# Self-Assembly at Temperature 1 Tiling the Plane 

Tile set:


If tiles abut with matching kinds of (positive strength) glue, then they bind.

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Self-Assembly at Temperature 1 Tiling the Plane

## Self-Assembly at Temperature 1 Building a Periodic Line

Tile set:


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Tile set:


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Tile set:


## Self-Assembly at Temperature 1 Building a Periodic Line

Tile set:

$\square$

## Self-Assembly at Temperature 1 Building a Periodic Line

Tile set:


| S 1 | $11^{2}$ | 221 | 1 | 1 | 2 | 2 | 2 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

## Self-Assembly at Temperature 1 Building a Periodic Line

Tile set:



## Self-Assembly at Temperature 1 Building a "Comb"

Tile set:

| 3 |
| :--- |
| 4 |
| 4 |


| 4 |
| :--- |
| 3 |
| 3 |



## Self-Assembly at Temperature 1 Building a "Comb"

Tile set:

| 3 |
| :--- |
| 4 |
| 4 |


| 4 |
| :--- |
| 3 |
| 3 |



## Self-Assembly at Temperature 1 Building a "Comb"

Tile set:

| 3 |
| :--- |
| 4 |
| 4 |


| 4 |
| :--- |
| 3 |
| 3 |



| S | 1 | 1 | 1 | 2 | 2 | 2 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Self-Assembly at Temperature 1 Building a "Comb"

Tile set:

| 3 |
| :--- |
| 4 |
| 4 |


| 4 |
| :--- |
| 3 |
| 3 |



## Self-Assembly at Temperature 1 Building a "Comb"

Tile set:

| 3 |
| :--- |
| 4 |
| 4 |


| 4 |
| :--- |
| 3 |
| 3 |



## Self-Assembly at Temperature 1 Building a "Comb"

Tile set:

| 3 |
| :--- |
| 4 |
| 4 |


| 4 |
| :--- |
| 3 |
| 3 |



# Self-Assembly at Temperature 1 Building a "Comb" 

Tile set:

| 3 |
| :--- |
| 4 |
| 4 |


| 4 |
| :--- |
| 3 |
| 3 |



# Self-Assembly at Temperature 1 Building a "Comb" 

Tile set:

| 3 |
| :--- |
| 4 |
| 4 |


| 4 |
| :--- |
| 3 |
| 3 |

$\left[\begin{array}{ll}\mathrm{S} & 1\end{array}\right]\left[\begin{array}{lll}3 & 3 \\ 1 & 1 & 2\end{array}\right]\left[\begin{array}{lll}2 & 2 & 1\end{array}\right]$


# Self-Assembly at Temperature 1 Building a "Comb" 

Tile set:

| 3 |
| :--- |
| 4 |
| 4 |


| 4 |
| :--- |
| 3 |
| 3 |



# Self-Assembly at Temperature 1 Building a "Comb" 

Tile set:

| 3 |
| :--- |
| 4 |
| 4 |


| 4 |
| :--- |
| 3 |
| 3 |

$\left[\begin{array}{rr}\mathrm{S} & 1\end{array}\right]\left[\begin{array}{lll}3 & 3 \\ 1 & 1 & 2\end{array}\right]\left[\begin{array}{lll}2 & 2 & 1\end{array}\right]$


# Self-Assembly at Temperature 1 Building a "Comb" 



## Self-Assembly at Temperature 1 Building an "Eventual Comb"

## Self-Assembly at Temperature 1 Building an "Eventual Comb"



## Self-Assembly at Temperature 1 Building an "Eventual Comb"



Self-Assembly at Temperature 1 Building an "Eventual Comb"


# Self-Assembly at Temperature 1 Building a Plane-Filling Grid 

# Self-Assembly at Temperature 1 Building a Plane-Filling Grid 



# Self-Assembly at Temperature 1 Building a Plane-Filling Grid 



# Self-Assembly at Temperature 1 Building a Plane-Filling Grid 



## Self-Assembly at Temperature 1 Building a Plane-Filling Grid

## Power of Cooperative Binding

- "Temperature 1" self-assembly: any tile may bind if even one glue matches


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## Power of Cooperative Binding

nperature 1" self-assembly: any t"~....ay bir l if even one glue matches

Pe ve or corme atre ering ing
"He np ratu e 1 elf asse nbl, an t", may bir lif $\epsilon$ en le lue nat es

Pe ve or comot atre ering ing
"He np ratu e 1 elf asse nbl, an t",...ay bir lif $\epsilon$ en lue lue nat es

## Pc ve or conoe atr e Bincing

- "n.np ratu e 1 elf asse nbl, an t"... ay bir lif $\epsilon$ en lue lue nat es


## Pumpability

Our result relies on a notion of pumpable paths in an assembly.

Partial assembly, with grey seed tile


## Pumpability

Our result relies on a notion of pumpable paths in an assembly.


## Pumpability

## Our result relies on a notion of pumpable paths in an assembly.

Look at just this partial path


## Pumpability

Our result relies on a notion of pumpable paths in an assembly.

Uniquely color each tile type


## Pumpability

Our result relies on a notion of pumpable paths in an assembly.
Note the repeating pattern, beginning with yellow tiles, which we call a pumpable segment


## Pumpability

Our result relies on a notion of pumpable paths in an assembly.
The pumpable segment can be infinitely repeated, or pumped, to create an infinite, periodic path


## Pumpable Tile Assembly System

A directed temperature 1 TAS $\mathbf{T}$ is $c$-pumpable if, given any two points $p$ and $q$ at least distance $c$ apart in the terminal assembly of $\mathbf{T}$, there is a path from $p$ to $q$ that contains a pumpable segment within the first $c$ points on the path.

In other words, every long path contains repetitions of a tile type (an obvious consequence of the pigeonhole principle) that can be pumped (repeated infinitely many times) to create a periodic path without colliding with the assembly up to that point.

## Pumpability

Not all repeating patterns are pumpable!


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## Pumpability

Not all repeating patterns are pumpable!


## Linear Sets

A set of points is linear if every point in the set can be expressed as a nonnegative integer affine combination of two integer vectors:

The point $b+3 u+2 v$
The dark green tiles make up a linear set


An initial offset $b$ from the origin, plus a multiple of $u$ plus a multiple of $v$

