CS 222 Winter 2011 HW 6 Due Thursday Feb. 24

1. Let G be a directed graph with designated nodes s and t. A set of paths from s to t in G are *node disjoint* if the only nodes they share are s and t. Prove that the maximum number of node disjoint s,t paths in G is equal to the minimum number of nodes needed to remove to disrupt all s,t paths in G.

Hint: Show that the problem of finding a maximum number of node disjoint s,t path in G, and the problem of finding the minimum number of nodes needed to remove to disrupt all s,t paths in G can be cast as a problem of finding a max flow and a min cut in some graph derived from G. Justify your answer. It may help to read the analogous result (7.45) in the book concerning edge disjoint paths in section 7.6.

2. Suppose we have a binary matrix M, i.e., one with 0 and 1 entries. We want to find the minimum number of rows and columns to delete so that every entry of value 1 is removed from M. Show how to solve this problem as a problem of computing a minimum s,t cut in some directed graph. Justify your solution.

3. In our exposition of the Ford-Fulkerson algorithm we described how to find a minimum cut at the end of the algorithm, i.e., when a flow f has been computed and there is no s to t path in the residual graph G_f. The minimum s,t cut was defined as the (A,B) node partition where A is the set of nodes reachable from s in G_f and $B = V - A$. We showed that (A,B) is indeed a minimum capacity s,t cut. This is also proven in the book as statement 7.9. Review that if needed.

Now consider a different s,t partition (P,Q) where Q is defined as the set of all nodes in G_f that can reach t. That is, a node v is in Q if and only if it can reach t via some directed path in G_f. Then $P = V - Q$. Prove that (P,Q) is a minimum capacity s,t cut. Model this proof after the proof that (A,B) is a minimum capacity cut.

Now prove that $A \subseteq P$. What is the relationship of B and Q?