Midterm Exam

Instructions:

• Please write neatly: if I can't read it, it isn't right.
• The exam is closed-book, closed-notes, closed-devices, closed-neighbors. Anyone violating these rules will be fed to a komodo dragon kept just for this purpose.
• If I speak of an "efficient" procedure, I mean this loosely: something that runs in polynomial, and not exponential, time.
• Good luck and kind wishes,

Phillip Rogaway

Name:

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1 Fill in the Blank

Complete the following narrative, following the conventions of lecture and your text.

1. A **DFA** $M = (Q, \Sigma, \delta, q_0, F)$ has $|Q| = 10$ states and $|\Sigma| = 2$ characters. Then there are [ ] points in the domain of δ, and [ ] points in the range of δ. *Answers are numbers.*

An **NFA** $M = (Q, \Sigma, \delta, q_0, F)$ has $|Q| = 10$ states and $|\Sigma| = 2$ characters. Then there are [ ] points in the domain of δ, and [ ] points in the range of δ. *Answers are numbers.*

2. Every NFA-acceptable language is DFA-acceptable. To prove this, suppose you have an NFA $M = (Q, \Sigma, \delta, q_0, F)$. For simplicity, let's assume that M has no ε-arrows: $\delta(q, \varepsilon) = \emptyset$ for all $q \in Q$. We must convert the NFA $M = (Q, \Sigma, \delta, q_0, F)$ into a DFA $M' = (Q', \Sigma, \delta', q_0', F')$ for the same language. As an example, if the NFA M has 10 states, 3 of them final, then the DFA M' we build from it will have $|Q'| =$ [ ] states, of which $|F'| =$ [ ] will be final. *Answers are numbers.*

3. You are given a first DFA $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ with $|Q| = 10$ states, $|F| = 5$ of them final. You are given a second DFA $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ with $|Q| = 10$ states, $|F| = 5$ of them final. Suppose you use the product construction to make a DFA $M = (Q, \Sigma, \delta, s, F)$ for $L(M_1) \cup L(M_2)$. It will have $|Q| =$ [ ] states and $|F| =$ [ ] of them will be final. *Answers are numbers.*

4. Suppose you convert $(00 \cup 11)^*$ into an NFA M using the procedures shown in class and in the book. Then M will have [ ] states. *The answer is a number.*

5. In class we described how you can convert a DFA $M = (Q, \Sigma, \delta, q_0, F)$ into a context-free grammar $G = (V, \Sigma, R, S)$ for the same language. Our grammar will have $|V| =$ [ ] variables and $|R| =$ [ ] rules. *Answers are formulas involving the components of the DFA.*

2 Short Answer

1. Draw the smallest (=fewest state) **DFA** possible for the language $L = 0 \cup (111)^*$. *Be careful; probably no partial credit. Make sure to include all necessary transitions and to to mark your start state final state(s).*

2. Suppose you want to prove that there is no smaller DFA for $L = 0 \cup (111)^*$ than the one you just gave. Give the first few sentences for such a proof. If your proof is going to involve examining multiple cases, identify them and then select **one** case to work out in full. *No credit if problem (1), just above, is wrong.*

3. Alice grades problem 1 (top of this page). She looks at Prof. Rogaway's key and marks as wrong any solution with a different number of final states. A student complains: "While my DFA has a different number of final states" he says, "it *too* is a smallest DFA for L; it just works differently." Could the student be right? Carefully explain.

4. Carefully state the **Pumping Lemma** for **regular languages**.

5. Use the pumping lemma to prove that $L = \{a^n \; b \; a^n : \; n \text{ is even}\}$ is **not** regular.

6. Give a CFG for the language $L = \{a^n \; b \; a^n : \; n \text{ is even}\}$. Your CFG should use at most one variable and two rules.

7. In a few clear sentences, describe the main idea behind the proof of the **Pumping Lemma** for **Context-Free Languages**. Draw a **picture** that helps to explain the main idea.

8. Prove or disprove: the CFLs are closed under intersection.

9. Let $L = \{x \in \{a,b,c\}^* : x$ contains exactly one a and exactly one $b\}$. Write a regular expression for this language Make it as short as possible.

10. For $a \in \Sigma$ and $x \in \Sigma^*$, let $d_a(x)$ be the string obtained by deleting each a from x (eg, $d_a(abbca) = bbc$ and $d_a(aaa) = \varepsilon$). Let $D_a(L) = \{d_a(x) : x \in L\}$. The CFLs are closed under D_a. Explain why.

11. In class we described the **CYK algorithm** to decide if a string x is in the language of a CFG G. In a cogent and well-written paragraph, sketch how this procedure works. *You should not write out detailed pseudocode; write out the sort of explanation you'd find in a well-written book describing the ideas underlying the algorithm.*

3 No-Justification True-False

Darken (completely fill in) the **correct** answer. If you don't know an answer, please **guess**.

1. **True** **False** If $M = (Q, \Sigma, \delta, q_0, F)$ is a DFA and $F = Q$ then $L(M) = \Sigma^*$.
2. **True** **False** If $M = (Q, \Sigma, \delta, q_0, F)$ is an NFA and $F = Q$ then $L(M) = \Sigma^*$.
3. **True** **False** Complementing the final state set of an NFA M gives an NFA for $\overline{L(M)}$.
4. **True** **False** For a DFA $M = (Q, \Sigma, \delta, q_0, F)$, we let δ^* be the Kleene-closure of δ, $\delta = \bigcup_{i \geq 0} \delta^i$.
5. **True** **False** If L is regular then L^* is regular.
6. **True** **False** If $L \subseteq \Sigma^*$ for an alphabet Σ, then L is regular iff $L \cup \Sigma$ is regular.
7. **True** **False** If $L \subseteq \Sigma^*$ for an alphabet Σ, then $(L \cup \Sigma)^*$ is regular.
8. **True** **False** If L is cofinite, meaning that its complement is finite, then L is regular.
9. **True** **False** The intersection of a finite language and an arbitrary language is regular.
10. **True** **False** The union of an infinite number of regular languages is regular.
11. **True** **False** Every subset of a regular language is regular.
12. **True** **False** A regular expression α is a string.
13. **True** **False** The pumping lemma is a useful tool for proving languages regular.
14. **True** **False** $\emptyset^* = \emptyset$.
15. **True** **False** $L = \{w \in \{0,1\}^* : w$ contains an equal number of 01's and 10's$\}$ is regular.
16. **True** **False** The intersection of a CFL and a regular language is context free.
17. **True** **False** $\{a^{2^n} : n \geq 0\}$ is regular.
18. **True** **False** If a language L is not regular, this can always be demonstrated by using the pumping lemma.
19. **True** **False** If A and B agree on all but a finite number of strings, then one is context free iff the other is.
20. **True** **False** An efficient algorithm is known that takes a regular expression α and a word w and decides if $w \in L(\alpha)$.
21. **True** **False** An efficient algorithm is known that takes in two DFAs and decides if they accept the same language.
22. **True** **False** Every regular language can be accepted by an NFA with only one final state.
23. **True** **False** There is a language L for which $L = L^*$.
24. **True** **False** If $G = (V, \Sigma, R, S)$ is a CFG, $L(G) \neq \emptyset$, and $S \to S \in R$, then G is ambiguous.
25. **True** **False** There is a CFL L such that G is ambiguous whenever $L = L(G)$.