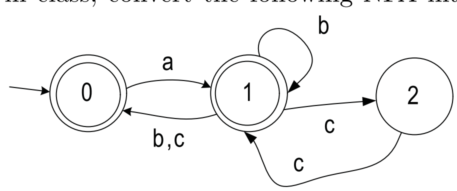
Problem Set 3 – Due Thursday, October 18, 2012

As a convenience for you, I will henceforth allow students to hand in their problem-set solutions in class, **as long as I have them before I start lecturing at 10:30 am**. Once I begin lecturing, no homework solution will be accepted. (The "nominal" way for you to turn in your problem set will remain to turn it in in the box in Kemper by 10:10 am.)

Problem 1. For the following problems, do not "simplify" your work (except you should please not indicate unreachable states in any DFA)—show everything.

(a) Using the procedure shown in class, convert the following NFA into a DFA for the same language.



(b) Using the procedure shown in class, convert the same NFA into a regular expression for the same language.

(c) Using the procedure shown in class, convert the following regular expression into an NFA for the same language:

$$(\text{big}^* \cup \text{bug})^*$$

Problem 2. Let $M = (Q, \Sigma, \delta, q_0, F)$ be an NFA. Inductively define the function $\delta^* : Q \times \Sigma^* \to \mathcal{P}(Q)$ that extends δ and has semantics "$\delta^*(q, x)$ is the set of all states reachable from q along x-labeled paths."

Problem 3. Let $L_n = \{1^i : 0 \le i < n\}$ (recall that $1^0 = \varepsilon$). Prove that there is a DFA M having n accepting states that accepts L. (Assume an underlying alphabet of $\Sigma = \{1\}$.) Then prove that L cannot be accepted by any DFA having fewer accepting states.

Problem 4 Construct a family of languages $\mathcal{L} = \{L_n\}$ $(n \ge 1)$ such that (1) the smallest NFA for L_n has $O(n)$ states, but (2) the smallest DFA for L_n has $\Omega(2^n)$ states. Prove that $\mathcal{L}$ this property.

Problem 5 Let $\text{Dbl}(L) = \{a_1 a_1 a_2 a_2 \cdots a_n a_n \in \Sigma^* : a_1 a_2 \cdots a_n \in L\}$. Prove that the DFA-acceptable languages are closed Dbl.