Problem Set 5 – Due Thursday, November 1, 2012

Problem 1. Formally specify both **(a)** a CFG and **(b)** a PDA for the language

$$L = \{x \in \{a, b, c\}^* : x \text{ contains an equal number of two different characters}\}.$$

Make your CFG and PDA as simple as possible. (If they ain't obviously right, they ain't right!)

Problem 2. Let $L = \{C \vdash C \vdash \cdots \vdash C \ : C \in \{0,1\}^*\}$. Convincingly argue that $\overline{L}$, the complement of L, is context free.

Problem 3. Consider the following CFG $G = (V, \Sigma, R, \text{STMT})$:

STMT → ASSIGN | IFTHEN | IFTHENELSE
IFTHEN → **if condition then** STMT
IFTHENELSE → **if condition then** STMT **else** STMT
ASSIGN → **a:=1**

with V being the variables in CAPS and Σ being the tokens in **bold**. We explained in class why G (or something just like it) is ambiguous. Provide an unambiguous CFG G', the simplest you can find, where $L(G') = L(G)$. Explain why G' is unambiguous.

Problem 4. Let $h\colon \Sigma_\varepsilon \to \Gamma^*$ be an arbitrary function. Extend it to strings and then languages by way of $h(a_1 a_2 \cdots a_n) = h(a_1)h(a_2)\cdots h(a_n)$ and then $h(L) = \{h(x)\colon x \in L\}$. (Here $a_1, \ldots, a_n \in \Sigma$ and $L \subseteq \Sigma^*$.) Prove: if L is context free then $h(L)$ is context free.

Problem 5. Let $G = (V, \Sigma, R, S)$ be a grammar in Chomsky Normal Form (CNF), $\varepsilon \notin L(G)$: every rule R of G has the form $A \to BC$ or $A \to a$ (for some $A, B, C \in V$, $a \in \Sigma$). Describe a (slow, but conceptually simple) decision procedure to answer the following decision question: *given $x \in \Sigma^*$, is $x \in L(G)$?* What is the running time of your algorithm in terms of $n = |x|$, $\nu = |V|$, $c = |\Sigma|$, and $r = |R|$?