Midterm Exam

Instructions: This is a closed book, closed notes exam. Do all **3** problems. Do your best to communicate your ideas clearly and succinctly. Good luck. —Phil Rogaway

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Name:

On problem	you got
1	
2	
3	
Σ	

1 Short Answer

1.1 Draw a **DFA** M for the language

$$L = \{x \in \{a, b, c\}^* : x \text{ contains exactly one } a \text{ and exactly one } b\}.$$

Make your DFA have as few states as possible.

1.2 List the first five strings of this language (Problem 1.1) in lexicographic order. Assume $a < b < c$.

1.3 Write a regular expression for this language (Problem 1.1). Make it as short as possible.

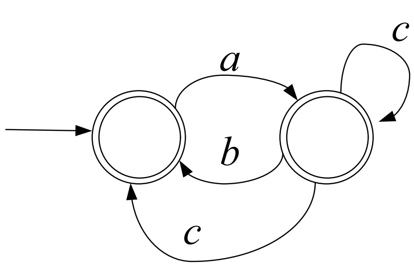
1.4 Give a CFG for $L = (\text{ab} \cup \text{aaa})^* \text{baa}$. Make your grammar use as few rules as possible.

1.5 Let $M = (Q, \Sigma, \delta, q_0, F)$ be an NFA with no ϵ-arrows. We can convert M into a DFA $M' = (Q', \Sigma, \delta', \{q_0\}, F')$ whose language is $L(M)$ by setting

$Q' =$ [ ] and $\delta'(S, a) =$ [ ]

and $F' = \{T \subseteq Q : T \cap F \neq \emptyset\}$.

1.6 Using the procedure shown in class, convert the following NFA into a regular expression for the same language.



2 Justified True or False

Put an **X** through the **correct** box. Then provide a brief justification. **Where appropriate, make the justification a counter-example.**

2.1 Every regular language can be accepted by an **NFA** with only a single final state.

Justification: **True** **False**

2.2 The complement of a regular language is context free. **True** **False**

Justification:

2.3 Let $h : \Sigma \to \Sigma^*$ be a function and define $h(a_1 \cdots a_n) = h(a_1) \cdots h(a_n)$ and $h(L) = \{h(x) : x \in L\}$. Suppose $h(L)$ is not regular. Then L is not regular.

Justification: **True** **False**

2.4 There is a language L for which $L = L^*$. **True** **False**

Justification:

2.5 Every nonempty regular language L is generated by some ambiguous CFG.

Justification: **True** **False**

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3 Classify

3.1. Let $L = \{ww : w \in \{0,1\}^*\}$. Is L regular? Prove your answer.

3.2. Let $L = \{w \in \{0,1\}^* : w \text{ contains an equal number of 01's and 10's}\}$. Is L regular? Prove your answer.