Problem Set 8

Problem 1.

Part A. Suppose that L is r.e. Show that L^* is r.e.

Part B. Suppose that L is recursive. Show that L^* is recursive.

Problem 2. Classify each of the following problems as either **decidable**—I see how to decide this language; **r.e.**—I don't see how to decide this language, but I can see a procedure to accept this language; **co-r.e.**—I don't see how to decide this language, but I can see a procedure to accept the complement of the language; **neither**: I don't see how to accept this language nor its complement.

Part A. $\{\langle M\rangle : M$ is a TM that accepts some string of prime length$\}$.

Part B. $\{\langle M\rangle : M$ is a C-program that halts on $\langle M\rangle\}$.

Part C. $\{\langle G\rangle : G$ is a CFG and G accepts an odd-length string$\}$.

Part D. $\{\langle M\rangle : M$ is a TM and M has 150 states$\}$.

Part E. $\{\langle M\rangle : M$ is a TM and $L(M) = L(M)^*\}$.

Part F. $\{\langle M\rangle : M$ is a TM and $L(M) = \emptyset\}$.

Part G. $\{\langle M\rangle : M$ is a TM and $L(M)$ is r.e. $\}$.

Part H. $\{\langle G_1, G_2\rangle : G_1$ and G_2 are CFGs and $L(G_1) = L(G_2)\}$.

Part I. $\{\langle M\rangle : M$ is a TM and M will visit state q_{25} when run on some input $x\}$.

Part J. $\{\langle M\rangle : M$ is a TM and M makes use of at most 50 tape cells when run on blank tape$\}$.

Problem 3. Recall that $L = \{ww : w \in \{a,b\}^*\}$ is not context free. Exhibit an unrestricted grammar for it. An unrestricted grammar is like a CFG except that rules can look like $aBCb \to accbDe$, for example: left-hand sides can contain any string of terminal and nonterminals with at least one nonterminal.