Midterm Exam

Instructions: The exam has six pages, not including this cover page. Read everything carefully. Write everything carefully, too; you will be graded on clarity *and* correctness. Please write neatly as well.

Relax. Breathe. The exam is not too long or too hard.

— Phil Rogaway

Name:

On page	you got
1	
2	
3	
4	
5	
6	
Σ	

1 Short Answer

(1.1) Complete the definition: A ***language*** over an alphabet Σ is

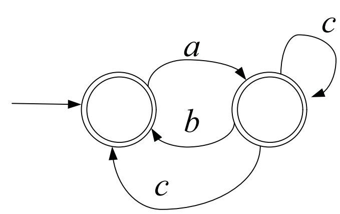
(1.2) List the first five strings, in lexicographic order, of the language

$$L = \{x \in \{a, b, c\}^* : x \text{ contains at least one } a \text{ and at least one } b\}.$$

Assume, as usual, that characters are ordered $a < b < c$.

(1.3) Explain what the **product construction** is and what we used it for.

(1.4) Using the procedure given in class and in your text, convert the following NFA M into a regular expression α such that $L(M) = L(\alpha)$. Do not "simplify" anything as you go.



(1.5) You are given the regular expression $\alpha = (00 \cup 11)^*$. *Composing the constructions given in class and in your text* (do not "simplify" anything), imagine converting α into a **DFA** M for which $L(M) = L(\alpha)$. How many states will M have? Justify your answer.

(1.6) Complete the definition: A CFL L is ***inherently ambiguous*** if: *(be precise with your quantifiers)*

(1.7) Complete the definition: A **regular** grammar is a CFG $G = (V, \Sigma, R, S)$ where:

(1.8) Carefully explain what it **means** if one says "the CFLs are closed under union." Then prove that this statement is true.

(1.9) Carefully state the ***pumping lemma*** for **context free** languages. (Don't use the word "pumps" without defining it.)

2 Justified True or False

Put an **X** through the **correct** box. When it says "Explain" provide a **brief** (but convincing) justification. **Where appropriate, make this justification a counterexample.** Choose the simplest counterexample you can find.

2.1. For every number n, the language $L_n = \{0^n 1^n\}$ is regular.

Explain: **True** **False**

2.2. If L^* is regular then L is regular.

Explain: **True** **False**

2.3. Let $M = (Q, \Sigma, \delta, q_0, F)$ be an **NFA** and let $M' = (Q, \Sigma, \delta, q_0, F')$, where $F' = Q \setminus F$ is the complement of F relative to Q. Then $L(M) = \overline{L(M')}$.

Explain: **True** **False**

2.4. Let $L = \{a^n b^n : n \geq 0\}$. Then $\overline{L}$ is regular.

Explain: **True** **False**

2.5. Let $\mathsf{noPrefix}(L) = \{w \in L |$ no proper prefix of w is in $L\}$. If L is regular, then so is $\mathsf{noPrefix}(L)$.

True **False**

Explain:

2.6. The pumping lemma is a useful technique to show that a language is regular.

Explain

True **False**

2.7. Language $L = \{w \in \{0,1\}^* :\ w$ has an equal number of 01's and 10's$\}$ is regular.

Explain

True **False**

2.8. If an **NFA** $M = (Q, \Sigma, \delta, q_0, F)$ has only accepting states (i.e., $F = Q$), then $L(M) = \Sigma^*$.

Explain:

True **False**

3 Simple Proofs

(3.1) Let $L = \{x \in \{a, b\}^* : \ |x| < 5\}$. Prove that there is a 6-state DFA that accepts L.

(3.2) Let $L = \{x \in \{a, b\}^* : \ |x| < 5\}$. Prove that there is **no** 5-state DFA that accepts L.

(3.3) Let $L = \{www : w \in \{a, b\}^*\}$. Prove that L is not regular.