Problem Set 2 – Due Tuesday, April 13, 2010, at 4:15 pm

Instructions: Write up your solutions as clearly and succinctly as you can. Typeset solutions, particularly in LaTeX, are always appreciated. Don't forget to acknowledge anyone with whom you discussed problems. Beginning with this homework, homeworks are to be due at **4:15 pm** (no longer 4:40 pm).

Problem 1. Let $\mathsf{canExtend}(L) = \{x \in L : \text{ there exists a } y \in \Sigma^+ \text{ for which } xy \in L\}$.

Part A. What is $\mathsf{canExtend}(\{\mathtt{0}, \mathtt{1}\}^*)$? What is $\mathsf{canExtend}(\{\varepsilon, 0, 1, 00, 01, 111, 1110, 1111\})$?

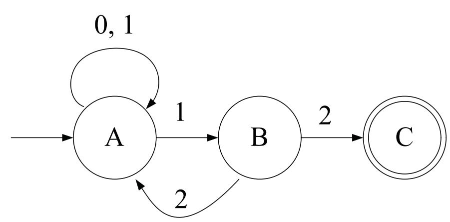
Part B. Prove that if L is DFA-acceptable then $\mathsf{canExtend}(L)$ is too.

A *prefix* of a string y is a string x such that $y = xx'$ for some x'. A prefix is *proper* if it is not the empty string. For any language L, let $\mathsf{noPrefix}(L) = \{w \in L | \text{ no proper prefix of } w \text{ is in } L\}$.

Part C. What is $\mathsf{noPrefix}(\{\mathtt{0}, \mathtt{1}\}^*)$? What is $\mathsf{noPrefix}(\{\varepsilon, 00, 01, 110, 0100, 0110, 1110, 1111\})$?

Part D. Prove that if L is DFA-acceptable then so is $\mathsf{noPrefix}(L)$.

Problem 2. Using the procedure shown in class, convert the following NFA into a DFA for the same language.



Problem 3. let $L = \{1^i : 0 \leq i < 10\}$ (recall that $1^0 = \varepsilon$). Prove that there is a DFA M having 10 accepting states that accepts L. Then prove that L cannot be accepted by any DFA having fewer accepting states.

Problem 5. Consider applying the product construction to NFAs $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ in order to show that the NFA-acceptable languages are closed under intersection.

Part A. Formally specify the product machine $M = (Q, \Sigma, \delta, q_0, F)$.

Part B. Does the construction work—that is, is $L(M) = L(M_1) \cap L(M_2)$? Informally argue your conclusion.

Problem 5. Prove that the DFA-acceptable languages are closed under reversal.

Problem 6 Consider trying to show that the NFA-accpetable languages are closed under $*$ (Kleene closure) by way of the following construction: *add ε-arrows from every final state to the start state; then finalize the start state, too.* Show, by finding a small counterexample, that the proposed construction does not work.