Problem Set 9 – Due <u>Thursday</u>, June 3, 2010, at 4:15 pm

Note the atypical due date, due to the Monday holiday.

Problem 1. Let MCF = $\{\langle M \rangle : M$ is TM and $L(M)$ is context free$\}$. Show that MCF is neither r.e. nor co-r.e.

Problem 2. Suppose you are given a polynomial time algorithm DECIDE that, on input of a Boolean formula ϕ, decides if ϕ is satisfiable. Using this procedure, describe a polynomial-time procedure FIND that produces a satisfying assignment t for the satisfiable formula ϕ. How many calls to DECIDE does your FIND procedure make?

Problem 3. Let SAT7 = $\{\langle \phi \rangle \mid \phi$ is a boolean formula having at least **seven** satisfying assignments$\}$. Show that SAT7 is NP-complete.

Problem 4. A graph $G = (V, E)$ is said to be *k-colorable* if there is a way to paint its vertices using colors in $\{1, 2, \ldots, k\}$ such that no adjacent vertices are painted the same color. When $k \geq 1$ is a number, by kCOLOR we denote the language of (encodings of) k-colorable graphs. The language 3COLOR is NP-Complete. (You can assume this.) Show that 4COLOR is NP-Complete.

Problem 5* (extra credit). Let

DE = $\{\langle p \rangle : p$ is a polynomial (in any number of variables) and p has an integral root.$\}$

Show that DE is NP-hard. Is it NP-complete?