Quiz 1 Solutions

Questions 1–4 were graded out of 10 points; questions 5–13 were graded out of 25 points total; question 14 was graded out of 20 points. So the total possible was 85 points.

1. *Complete the following, being mathematically precise and following the conventions of your text: An NFA is a five-tuple $M = (Q, \Sigma, \delta, q_0, F)$ where: Q is a finite set, Σ is an alphabet, $q_0 \in Q$, $F \subseteq Q$, and δ is a function having domain and range*

$$\delta \colon Q \times (\Sigma \cup \{\varepsilon\}) \to \mathcal{P}(Q)$$

2. *Draw a **DFA** for the language:*

$$L = \{x \in \{a,b,c\}^* : x \text{ contains at least one } a \text{ and at least one } b\}$$

Make your DFA use a minimal possible number of states. Don't forget the c's.

3. *List the first five strings, according to the lexicographic ordering,[1] of the language*

$$L = \{x \in \{a,b,c\}^* : x \text{ contains at least one } a \text{ and at least one } b\}.$$

ab, ba, aab, aba, abb

[1] Recall that in the lexicographic ordering of a language L we first list all length-0 stings in L, then all length-1 strings in L, then all length-2 strings in L, and so on. Among strings of a given length, we use a fixed ordering of the characters. You should here assume an ordering of characters $a < b < c$.

4. *Write the simplest regular expression you can for the language*

   $$L = \{x \in \{0,1,2\}^* : x \text{ contains exactly two '2's}\}$$

   $(0 \cup 1)^* 2 (0 \cup 1)^* 2 (0 \cup 1)^*$

5. *True or* ***False****: If there's a 10-state DFA that accepts L_1 and there's a 10-state DFA that accepts L_2, then there's a 20-state DFA that accepts $L_1 \cap L_2$.*

6. ***True*** *or False: If there's a 10-state DFA that accepts L_1 and there's a 10-state DFA that accepts L_2, then there's a 100-state DFA that accepts $L_1 \cap L_2$.*

7. ***True*** *or False: If there's a 10-state DFA that accepts L_1 and there's a 10-state DFA that accepts L_2, then there's a 200-state DFA that accepts $L_1 \cap L_2$.*

8. ***True*** *or False: Every NFA-acceptable language is DFA-acceptable.*

9. *True or* ***False****: If M is an NFA and $x \in L(M)$ then every x-labeled path in the diagram of M that starts at the start state of M ends in an accept state of M.*

10. *True or* ***False****: An alphabet can be infinite or finite.*

11. *True or* ***False****: The complement of an infinite language is finite.*

12. *True or* ***False****: The concatenation of an infinite language and a finite language is always infinite.*

13. *True or* ***False****: The Kleene-closure of a language (that is, a language L^*) is always infinite.*

14. *Carefully explain what it* ***means*** *when we say "the DFA-acceptable languages are closed under complement." Then explain* ***why*** *this statement is true.*

    The statement means that if L is DFA-acceptable then so is $\overline{L}$, the complement of L with respect to the underlying alphabet. Said differently, if there exists a DFA that accepts L then there exists a DFA that accepts $\overline{L}$. The reason these statements are true is that, given a DFA $M = (Q, \Sigma, \delta, q_0, F)$ for L, a DFA M' for $\overline{L(M)}$ can be constructed by way of $M' = (Q, \Sigma, \delta, q_0, F')$ where $F' = Q \setminus F$. It is immediate that $x \in L(M)$ iff $x \notin L(M')$, so $L(M') = \overline{L(M)}$.