Quiz 4

Neatly print: **Firstname LASTNAME:**

Seat number:

Instructions: TM = Turing machine. $A_{\rm TM} = \{\langle M, w\rangle :\ \text{TM } M \text{ accepts } w\}$. Turing acceptable = r.e. = recursively enumerable. Recursive = Decidable. A language is co-r.e. if its complement is r.e. Write $A \le_{\rm m} B$ for A many-one-reduces to B. No notes/books/gadgets/neighbors. Be mathematically precise.

1. Explain the difference between
   — a TM M **deciding** a language L, and
   — a TM M **accepting** a language L.

2. State the **Church-Turing Thesis**:

3. Let A and B be languages. Then we say that $A \le_{\rm m} B$ if:

   Do not give me an English-language description—I am asking for a rigorous definition.

4. Let FINITE $= \{\langle M\rangle :\ M \text{ is a TM and } L(M) \text{ is finite}\}$.
   Complete the following proof that $A_{\rm TM} \le_{\rm m}$ FINITE:

   We must give a Turing-computable function f to map $\langle M, w\rangle$ to $\langle M'\rangle$ such that M accepts $w \iff L(M')$ is finite.
   To accomplish this, we will have
   M', on input x: *Describe here how M' works in three lines of pseudocode.*

   Now if M accepts w then $L(M') = \Sigma^{<N}$, where N is the number of steps it takes M to accept w. This language is a finite set. On the other other hand, if M does not accept w, then $L(M') = \Sigma^*$, which is infinite.

5. Classify each of the following languages as either **decidable**—I see how to decide this language; **r.e.**—I don't see how to decide this language, but I can see a procedure to accept it; **co-r.e.**—I don't see how to decide this language, but I can see a procedure to accept its complement; or **neither**: I don't see how to accept this language or its complement. No justification is needed for your answers.

☐ (a) $\{\langle M \rangle : M \text{ is a TM that accepts some string of prime length}\}$.

☐ (b) $\{\langle M \rangle : M \text{ is a TM and } M \text{ has 100 states}\}$.

☐ (c) $\{\langle M \rangle : M \text{ is a TM and } L(M) \text{ is regular}\}$.

☐ (d) $\{\langle M \rangle : M \text{ is a TM and } L(M) \text{ is r.e.}\}$.

☐ (e) $\{x : x \text{ is a C-program (no I/O or library calls) that halts on } x\}$.

☐ (f) $\{\langle M \rangle : M \text{ is a TM and } M \text{ will visit state } q_{20} \text{ when run on some input } x\}$.

☐ (g) $\{\langle G \rangle : G \text{ is a CFG and } G \text{ accepts an odd-length string}\}$.

6. Darken the **correct** box. No justification is required. If you're not sure, guess.

(a) **True** **False** If TM M decides L, then M accepts L.

(b) **True** **False** $\{\langle G \rangle : G \text{ is an ambiguous CFG}\}$ is r.e.

(c) **True** **False** If TM M accepts L and M loops on some string, then L is not decidable.

(d) **True** **False** If L_1 is r.e. and L_2 is r.e. then $L_1 \cup L_2$ is r.e.

(e) **True** **False** If a language is r.e. and its complement is r.e., then the language is decidable.

(f) **True** **False** The language $\{\langle M \rangle : M \text{ is a TM and } L(M) \text{ is finite}\}$ is r.e.

(g) **True** **False** If L is context free then $\overline{L}$ is decidable.

(h) **True** **False** The union of infinitely many decidable languages is decidable.

(i) **True** **False** If $A_{\text{TM}} \leq_{\text{m}} B$ then B is r.e.

(j) **True** **False** If nondeterministic TM M can, on input x, get to *some* accepting configuration and *some* rejecting configuration, then M accepts x.

Done! Finished! Completed! Concluded!