Problem Set 2 – Due Friday, April 10, 2015

Problem 1 Draw DFAs for the following languages:

(a) $A = \{x \in \{a, b\}^* : \ |x| \geq 3\}$

(b) $B =$ the binary encodings of numbers divisible by 7. Allow leading zeros and the empty string as alternate names of 0. Thus $B = \{\varepsilon, 0, 00, 000, 111, 0000, 1110, \ldots\}$

(c) $C =$ the binary encodings of numbers divisible by 7. Don't allow leading zeros or the empty string. Thus $C = \{0, 111, 1110, \ldots\}$.

(d) $D =$ binary strings that contain the same number of 01's as 10's.

Problem 2. Let $\mathcal{E}(L) = \{x \in L : \text{ there exists a } y \in \Sigma^+ \text{ for which } xy \in L\}$. (By Σ^+ we mean $\Sigma\Sigma^*$.)

Part A. What is $\mathcal{E}(\{0,1\}^*)$? What is $\mathcal{E}(\{\varepsilon, 0, 1, 00, 01, 111, 1110, 1111\})$?

Part B. Prove that if L is DFA-acceptable then $\mathcal{E}(L)$ is, too.

Problem 3 State whether the following propositions are true or false, proving each answer.

(a) Every DFA-acceptable language can be accepted by a DFA with an odd number of states.

(b) Every DFA-acceptable language can be accepted by a DFA whose start state is never visited twice.

(c) Every DFA-acceptable language can be accepted by a DFA no state of which is ever visited more than once.

(d) The language $L = \{x \in \{a, b\}^* : x \text{ starts and ends with the same character}\}$ can be accepted by a DFA $M = (Q, \Sigma, \delta, q_0, F)$ for which $\delta^*(q_0, w) = q_0$ for some $w \neq \varepsilon$. Assume an alphabet of $\Sigma = \{a, b\}$.

Problem 4 A *homomorphism* is a function $h : \Sigma \to \Gamma^*$ for alphabets Σ, Γ. Given a homomorphism h, extend it to strings and then languages by asserting that $h(\varepsilon) = \varepsilon$, $h(a_1 \cdots a_n) = h(a_1) \cdots h(a_n)$ (for $a_1, \ldots, a_n \in \Sigma$), and $h(L) = \{h(x) : x \in L\}$.

(a) Prove: for any homomorphism h, if L is DFA-acceptable, then so is $h(L)$.

(b) Disprove: for any homomorphism h, if $h(L)$ is DFA-acceptable, then so is L. For this you may assume that there's a language L that is not DFA-acceptable.

Problem 5. Fix a DFA $M = (Q, \Sigma, \delta, q_0, F)$. For any two states $q, q' \in Q$, let us say that q and q' are *equivalent*, written $q \sim q'$, if, for all $w \in \Sigma^*$ we have that $\delta^*(q, w) \in F \Leftrightarrow \delta^*(q', w) \in F$. Here δ^* is the extension of δ to Σ^* defined by $\delta^*(q, \varepsilon) = q$ and $\delta^*(q, ax) = \delta^*(\delta(q, a), x)$.

(a) Prove that $\sim$ is an equivalence relation.

(b) Suppose that $q \sim q'$ for distinct q, q'. Describe, first in plain English and then in precise mathematical terms, how to construct a smaller (=fewer state) DFA M' that accepts the same language as M.