Problem Set 9 — Due Thusday, March 7, 2002

Problem 1 Prove that L is decidable iff L is listable in lexicographic order. (A language is listable in lexicographic order if some program outputs $x_1\sharp x_2\sharp x_3\sharp\cdots$, $L = \{x_1, x_2, x_3, \ldots\}$, and $x_1 < x_2 < x_3 < \cdots$ where "$<$" denotes the usual lexicographic ordering on strings.)

Problem 2 *(Counts as 20 points, same as 2 ordinary problems.)*

Part A. Let $L = \{\langle M\rangle : M \text{ is a TM that accepts some string of prime length}\}$. Prove that L is not decidable.

Part B. Let $L = \{\langle G\rangle : G \text{ is a CFG and } G \text{ accepts an odd-length string}\}$. Prove that L is not decidable.

Part C. Let $L = \{\langle M\rangle : M \text{ is a TM and } L(M) = L(M)^*\}$. Prove that L is not r.e.

Part D. Let $L = \{\langle M\rangle : M \text{ is a TM and } L(M) = L(M)^*\}$. Prove that L is not r.e.

Part E. Let $L = \{\langle M\rangle : M \text{ is a TM and } L(M) = L(M)^*\}$. Prove that L is not co-r.e.

Part F. Let $L = \{\langle G_1, G_2\rangle : G_1 \text{ and } G_2 \text{ are CFGs and } L(G_1) = L(G_2)\}$. Prove that L is not decidable. You may use the fact that $A = \{\langle G\rangle : G \text{ is a CFG and } L(G) = \Sigma^*\}$ is undecidable.