Quiz — Section 1

Instructions: Please answer the questions succinctly and thoughtfully. Good luck.
— Phil Rogaway

Name:

Signature:

On problem	you got	out of
1		30
2		40
3		20
Σ		90

1 Short Answer [30 points]

(1) Draw a DFA that accepts $L = \{x \in \{1,2\}^* : \ x \text{ has exactly two 2's}\}$.

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(2) List the lexicographically-first six strings in the set $\{0, 10\}^*$. (Lexicographic order of $\{0,1\}^*$ is $\{\varepsilon, 0, 1, 00, 01, 10, 11, 000, \ldots\}$.)

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(3) Give a smallest DFA that accepts $\{\varepsilon\}$. The alphabet is $\Sigma = \{a\}$.

(4) Using the procedure we have seen in class ("follow the character and then chase down ε-arrows"), convert the following NFA into a DFA that accepts the same language.

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(5) Define what is a **language** over an alphabet Σ.

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(6) Recall that, for $L \subseteq \{0, 1\}^*$, $\text{PAL}(L) = \{x \in \{0, 1\}^* : xx^R \in L\}$. Write a regular expression for $\text{PAL}(\{0, 1\}^*)$.

2 Justified True or False [40 points]

Put an **X** through the **correct** box. Where it says "Explain" provide a **brief** (but convincing) justification. No credit will be given to correct answers that lack a proper justification. Where appropriate, **make your justification a counter-example**. Throughout, we use L to denote a language (maybe regular, maybe not).

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1. $\emptyset^* = \emptyset$ **True** **False**

Explain:

2. Every subset of a DFA-acceptable language is DFA-acceptable.

Explain: **True** **False**

3. If $M = (Q, \Sigma, \delta, q_0, F)$ is a DFA and $F = Q$ then $L(M) = \Sigma^*$. **True** **False**

Explain:

4. If L is accepted by an n-state NFA then L is accepted by some 3^n-state DFA.

Explain: **True** **False**

5. If $M = (Q, \Sigma, \delta, q_0, F)$ is an NFA and $M' = (Q, \Sigma, \delta, q_0, F')$, where $F' = Q - F$, then $L(M') = \overline{L(M)}$. **True** **False**

Explain:

6. If L is a not-regular language and F is a finite language then $L \cap F$ is a regular language. **True** **False**

Explain:

7. $(L^*)^* = L^*$. **True** **False**

Explain:

8. For α a regular expression, there is an algorithm to decide if $x \in L(\alpha)$ that is efficient enough to run in a reasonable amount of time on reasonable length x, α.

Explain: **True** **False**

3 A Closure Property of Regular Languages [20 points]

If L is a language over an alphabet Σ let

$$\text{NoPrefix}(L) = \{x \in L : \text{no proper prefix of } x \text{ is in } L\}.$$

Prove that if L is regular then so is $\text{NoPrefix}(L)$. *(Describe any construction you use both in clear English and by a formal definition.)*