Quiz — Section 2

Instructions: Please answer the questions succinctly and thoughtfully. Good luck.
— Phil Rogaway

Name:

Signature:

On problem	you got	out of
1		30
2		40
3		20
Σ		90

1 Short Answer [30 points]

(1) Draw a DFA that accepts $L = \{x \in \{1,2\}^* : \ x \text{ has at least two 2's}\}$.

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(2) List the lexicographically-first six strings in the complement of $\{b, aa, ab, aaa\}$. (Lexicographic order of $\{a,b\}^*$ is $\{\varepsilon, a, b, aa, ab, ba, bb, aaa, \ldots\}$.)

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(3) Give a smallest NFA that accepts $\{\varepsilon\}$. The alphabet is $\Sigma = \{a\}$.

(4) Using the procedure we have seen in class, convert the following NFA into a regular expression for the same language.

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(5) Define what is an **alphabet**.

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(6) Recall that, for $L \subseteq \{\mathtt{0}, \mathtt{1}\}^*$, $\mathrm{PAL}(L) = \{x \in \{\mathtt{0}, \mathtt{1}\}^* : xx^R \in L\}$. Write a regular expression for $\mathrm{PAL}(0^*1^*)$.

2 Justified True or False [40 points]

Put an **X** through the **correct** box. Where it says "Explain" provide a **brief** (but convincing) justification. No credit will be given to correct answers that lack a proper justification. Where appropriate, **make your justification a counter-example**. Throughout, we use L to denote a language (maybe regular, maybe not).

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1. Every DFA-acceptable language can be accepted by a DFA with more accepting states than non-accepting states. **True** **False**

Explain:

2. L^* is infinite. **True** **False**

Explain:

3. If $M = (Q, \Sigma, \delta, q_0, F)$ is an NFA and $F = Q$ then $L(M) = \Sigma^*$

Explain: **True** **False**

4. If L is finite and $\sum_{x \in L} |x| = N$ then L can be accepted by an NFA having $N + 1$ states. **True** **False**

Explain:

5. Every co-finite language can be accepted by a DFA. **True** **False**

Explain:

6. There exists a language L such that L is nonempty, L is closed under concatenation, and L contains no string of even length. **True** **False**

Explain:

7. L^+ (that is, LL^*) does not contain the emptystring. **True** **False**

Explain

8. Let $M = (Q, \{0,1\}, \delta, q_0, F)$ be a DFA and let $L = L(M)$. Suppose $x01001 \in L$ and $y01001 \notin L$. Then it is possible that $\delta^*(q_0, x) = \delta^*(q_0, y)$. **True** **False**

Explain:

3 A Closure Property of Regular Languages [20 points]

If L is a language over an alphabet Σ let

$$\text{Late}(L) = \{x \in \Sigma^* : \text{for some } a \in \Sigma, \text{ string } ax \in L\}.$$

Prove that if L is regular then so is $\text{Late}(L)$. *(Describe any construction you use both in clear English and by a formal definition.)*