Midterm Exam

Instructions: This is a closed book, closed notes exam. Write on the exam. There are five numbered pages, 1–5. Do your best to communicate your ideas clearly and succinctly. Ask me if you don't understand what a problem means. Good luck!

— Phil Rogaway

Your Name:

On problem	you got	out of
You-wrote-your-name		3
1		42
2		40
3		15
All together now		100

1 Short Answer [42 points]

1.1. Give a mathematically-precise translation of the following claim:

The context-free languages are closed under intersection.

Don't explain if the statement is true or false—just tell me what the statement *means*.

1.2. Draw a **DFA** that accepts $\overline{a^*b^*c^*}$. The complement is relative to the alphabet $\Sigma = \{a, b, c\}$. Make your DFA as small as you can.

1.3. Prove that the grammar $S \to aS \mid Sa \mid a$ is **ambiguous.**

1.4. List, in lexicographic order, the first **four** strings of $(22)^*(333)^*$.

1.5. Give a **CFG** for $L = \{a^n c^{n+2} :\ n \geq 0\}$. Make your grammar as simple as you can.

1.6. Prove that the language above, $L = \{a^n c^{n+2} :\ n \geq 0\}$, is **not regular**.

1.7. Describe an algorithm which, given a regular expression α and a number n, decides if $L(\alpha)$ contains a string of length at least n.

2 Justified True or False [40 points]

Put an **X** through the **correct** box. Also give a very **brief** (but convincing) justification (just a few words). Where appropriate, make the justification a counter-example.

2.1. If L is regular and F is finite then $L \oplus F$ is regular. (Recall that for sets A and B, by $A \oplus B$ means everything that is in exactly one of these two sets).

True **False**

2.2. If $L_1 L_2$ is regular then L_1 and L_2 are regular. **True** **False**

2.3. Every regular language can be accepted by an NFA that has exactly one final state.

True **False**

2.4. The intersection of a context free language and a regular language is context free.

True **False**

2.5. Suppose L is infinite and regular. Then there exists positive numbers a and b such that L contains some string of length $ai + b$ for every $i \geq 0$. **True** **False**

2.6. Every regular language L is generated by some non-ambiguous CFG.

True **False**

2.7. Given DFAs M_1 and M_2 with n_1 and n_2 states, respectively, there is a DFA M with $n_1 n_2$ states that accepts $L(M_1) \cap L(M_2)$. **True** **False**

2.8. Regular languages are closed under intersection and regular languages are context free, so context free languages are closed under intersection. **True** **False**

3 A Property of Regular Languages? [15 points]

For $x \in \{0,1\}^*$, let $\overline{x}$ denote the bitwise complement of x (e.g., $\overline{1011} = 0100$).

Given a language $L \subseteq \{0,1\}^*$, let $\mathcal{C}(L) = \{x \in \{0,1\}^* \colon \text{ both } x \text{ and } \overline{x} \text{ are in } L\}$.

If L is regular, is $\mathcal{C}(L)$ regular? Prove your answer.