Problem Set 1 — Due Tuesday, 11 January 2005

Instructions: Write up your solutions as clearly and succinctly as you can. Typeset solutions, particularly in LaTeX, are always appreciated. Don't forget to acknowledge anyone with whom you discussed problems. Recall that homeworks are due at 11 am on Tuesday in the turn-in box in Kemper, on the second floor.

Problem 1 A *T-string* is any of the following strings: the emptystring ε is a T-string; and if x and y are T-strings, then so are $1x0y0$ and $0x1y0$ and $0x0y1$. Prove or disprove the following: a string $x \in \{0,1\}^*$ is a T-string iff it has twice as many 0's as 1's.

Problem 2. For each of the following, give an example language to prove existence, or explain why there is no such example. Assume an underlying alphabet of $\{0,1\}$.

A. An infinite language L with an infinite complement.

B. A language L closed under concatenation and containing no even-length strings.

C. An infinite unary language L such that if $x \in L$ and $y \in L$ then there is no string in L of length $|x| + |y|$. (A unary language means that the underlying alphabet has just one character, say 1.)

D. A finite language L having a longest string x that is longer than a longest string of any other finite language.

E. Infinite languages L_1, L_2, L_3 with empty (three-way) intersection, but any two of which union to make $\{0,1\}^*$.

F. An infinite language L such that, for every number n, L contains strings x and X where $|X| - |x| > n$ and L contains no string y where $|x| < |y| < |X|$.