Problem Set 7 — Due March 1, 2005

Problem 7.1. Formally specify (draw a transition diagram for) a Turing machine that, when started on an initially empty, two-way infinite tape, will eventually visit any cell. Make your machine have as few states as you can. (You may lose points if your machine is more complicated than mine!)

Problem 7.2. Recall that an *unrestricted grammar* $G = (V, \Sigma, R, S)$ is just like a context-free grammar except that the rules are a finite subset of $(V \cup \Sigma)^* \; V \; (V \cup \Sigma)^* \times (V \cup \Sigma)^*$. Derivations in an unrestricted grammar are just like derivations in a CFG: if there is a rule $\alpha \to \beta$ and you see α in a sentential form, you can replace α by β (possibly resulting in the erasure or change of terminals). The language of G, $L(G)$ is the set of terminal strings derivable from the start symbol S.

Part A. Exhibit an unrestricted grammar for $L = \{ww : \; w \in \{a, b\}^*\}$

Part B. Prove that a language is r.e. if and only if it is generated by an unrestricted grammar.

Problem 7.3 (*Counts as two problems.*) Classify each of the following problems as either **decidable**—I see how to decide this language; **r.e.**—I don't see how to decide this language, but I can see a procedure to accept this language; **co-r.e.**—I don't see how to decide this language, but I can see a procedure to accept the complement of the language; **neither**: I don't see how to accept this language nor its complement.

Part A. $\{\langle M \rangle : M$ is a TM that accepts some palindrome$\}$.

Part B. $\{\langle M \rangle : M$ is a C-program that diverges on $\langle M \rangle\}$.

Part C. $\{\langle G \rangle : G$ is a CFG and G accepts an odd-length string$\}$.

Part D. $\{\langle M \rangle : M$ is a TM and $L(M) = L(M)^*\}$.

Part E. $\{\langle M \rangle : M$ is a TM and $L(M) = \emptyset\}$.

Part F. $\{\langle M \rangle : M$ is a TM and $L(M)$ is decidable$\}$.

Part G. $\{\langle G_1, G_2 \rangle : G_1$ and G_2 are CFGs and $L(G_1) = L(G_2)\}$.

Part H. $\{\langle M, x, q \rangle : M$ is a TM and M will visit state q when run on input $x\}$.

Part I. $\{\langle p \rangle : p$ is a multivariate polynomial and p has an integer root$\}$.

Part J. $\{\langle p \rangle : p$ is a monovariate polynomial and p has an integer root$\}$.