Problem Set 8 — Due March 8, 2005

Problem 8.1. Prove that a language is decidable iff it can be enumerated in lexicographic order. (A langugage L can be enumerated in lexicographic order if there is a TM that, on input of the empty string, produces an output of $\#x_1\#x_2\#x_3\#\cdots$ where $L = \{x_1, x_2, x_3, \ldots\}$ and $x_1 < x_2 < x_3 < \cdots$ under the lexicographic ordering of strings.)

Problem 8.2. (*Counts as two problems*)

A. Show that

$$L_A = \{\langle M, k\rangle : M \text{ is a TM which accepts at least one string of length } k\}$$

is not decidable.

B. Prove that

$$L_B = \{\langle M, k\rangle : M \text{ is a TM that loops on at least one string of length } k\}$$

is not decidable.

C. Let

$$\begin{aligned} L_C \;=\; & \{\langle M, k\rangle : M \text{ is a TM which accepts some string of length } k, \\ & \quad \text{but } M \text{ loops on some (other) string of length } k\}. \end{aligned}$$

Show that L_C is not r.e.. (Assume the underlying alphabet has at least two characters.)

D. Show that L_C is not co-r.e., either.

Problem 8.3. (*Counts as two problems*) Classify the following languages as **decidable**, **r.e.** (but not decidable), **co-r.e.** (but not decidable), or **neither** r.e. nor co-r.e.. Prove all your answers, giving decision procedures or reductions.

A. $L = \{\langle M\rangle : M$ accepts some even-length string$\}$.

B. $L = \{\langle M, w\rangle : M$ is a TM which uses at most 17 tape squares when run on $w\}$

C. $L = \{\langle M\rangle :\ M$ accepts some palindrome$\}$.

D. $L = \{\langle M\rangle :\ M$ never prints a "0" (regardless of the input)$\}$.

E. $L = \{\langle \alpha\rangle :\ \alpha$ is shortest regular expression for $L(\alpha)\}$.