# Problem Set 1 — Due Tuesday, January 10, 2006

**Instructions:** Write up your solutions as clearly and succinctly as you can. Typeset solutions, particularly in LaTeX, are always appreciated. Don't forget to acknowledge anyone with whom you discussed problems. Recall that homeworks are due at 1:15 pm on Tuesday in the turn-in box in Kemper Hall, room #2131.

**Problem 1.** We say that a string $x$ over an alphabet consisting of left parenthesis, '(', and right parenthesis, ')' is *1-balanced* if

a. $x$ has an equal number of left and right parentheses; and

b. any suffix of $x$ has at least as many right parentheses as left parentheses.

Say that a string $x$ (over the same alphabet) is *2-balanced* if it can be generated by the following rules:

a. The empty string, $\epsilon$, is 2-balanced.

b. If $x$ is 2-balanced, so is $(x)$.

c. If $x$ and $y$ are 2-balanced, so is $xy$.

d. Nothing else is 2-balanced.

**Part A.** Prove that if a string $x$ is 1-balanced, then it is 2-balanced. *Hint*: By induction on $|x|$.

**Part B.** Prove that if a string $x$ is 2-balanced, then it is 1-balanced. *Hint*: By induction on the definition of 2-balanced.

**Problem 2.** State whether the following propositions are true or false, explaining each answer.

**Part A.** $\emptyset$ is a language.

**Part B.** $\emptyset$ is a string.

**Part C.** $\epsilon$ is a language.

**Part D.** $\epsilon$ is a string.

**Part E.** Every language is infinite or has an infinite complement.

**Part F.** Some language is infinite and has an infinite complement.

**Part G.** The set of real numbers is a language.

**Part H.** There is a language that is a subset of every language.

**Part I.** The Kleene-star (Kleene closure) of a langauge is always infinite.

**Part J.** The concatentation of an infinite language and a finite language is always infinite.

**Part K.** There is an infinite language $L$ containing the emptystring and such that $L^i$ is a proper subset of $L^*$ for all $i \geq 0$.