Problem Set 10 — Due Tuesday, March 14, 2006

Problem 1. The following theorem was presented in class: A language L is decidable iff there exists an enumerator E that lists it in lexicographic order. Prove it.

Problem 2. Finish the proof of Rice's theorem in your handout by arguing the case when the emptyset does have property P.

Problem 3. Suppose you are given a polynomial time algorithm D that, on input of a Boolean formula ϕ, decides if ϕ is satisfiable. Describe an efficient procedure S that finds a satisfying assignment for ϕ. How many calls to D do you make?

Problem 4. Let $\mathit{MULT\text{-}SAT} = \{\langle\phi\rangle \mid \phi \text{ has at least ten satisfying assignments}\}$. Show that *MULT-SAT* is NP-complete.

Problem 5. A graph $G = (V, E)$ is said to be *k-colorable* if there is a way to paint its vertices using colors in $\{1, 2, \ldots, k\}$ such that no adjacent vertices are painted the same color. When k is a number, by $k\mathit{COLOR}$ we denote the language of (encodings of) k-colorable graphs. The language *3COLOR* is NP-Complete. (You can assume this.) Use this to prove that the language *4COLOR* is NP-Complete, too.

Problem 6. Let

$$D = \{\langle p\rangle : \ p \text{ is a polynomial (in any number of variables) and } p \text{ has an integral root.}\}$$

Prove that $\mathit{3SAT} \leq_{\mathrm{p}} D$.