Problem Set 7 — Due Tuesday, February 21, 2006

Problem 1. Show that the collection of decidable languages is closed under the operations of

Union.

Concatenation.

Star.

Complementation.

Intersection.

Problem 2. Show that the collection of Turing-recognizable languages is closed under the operations of

Union.

Concatenation.

Star.

Intersection.

Problem 3. Show that the problem of testing whether a CFG generates some string in 1^* is decidable. In other words, show that $\{\langle G \rangle \mid G \text{ is a CFG over } \{\mathtt{0},\mathtt{1}\}^* \text{ and } 1^* \cap L(G) \neq \emptyset\}$ is decidable.

Problem 4. Examine the formal definition of a Turing machine to answer the following questions, and explain your reasoning.

a. Can a TM ever write the blank symbol on its tape?

b. Can the tape alphabet Γ be the same as the input alphabet Σ?

c. Can a TMs head ever be in the same location in two successive steps?

d. Can a TM contain just a single state?

Problem 5. Give an implementation-level description of a Turing machine that decides $L = \{w \mid w \text{ contains an equal number of 0s and 1s}\}$ over the alphabet $\{0, 1\}$.

Problem 6. Say that a **write-once Turning machine** is a single-tape Turning machine that can alter each tape square at most once (including the input portion of the tape). Altering a tape square means overwriting the symbol it currently contains with some other symbol; merely moving over a tape square, reading and rewriting the symbol it contains, as when $\delta(q_i, 0) = (q_j, 0, R)$, is allowed.

Show that this variant Turing machine model is equivalent to the ordinary Turing machine model. (Hint: As a first step consider the case whereby the Turing machine may alter each tape square at most twice. Use lots of tape.)