Problem Set 8 — Due Thursday, March 15, at 1:00 pm

Problem 1. State whether the following claims are true or false, briefly explaining your answer.

a. $A \leq_{\mathrm{P}} A$.

b. If $A \leq_{\mathrm{P}} B$ and $B \leq_{\mathrm{P}} C$, then $A \leq_{\mathrm{P}} C$.

c. If $A \leq_{\mathrm{P}} B$ then $\overline{A} \leq_{\mathrm{P}} \overline{B}$.

d. If $A \leq_{\mathrm{P}} B$ and B is decidable then A is decidable.

e. If $A \in P$ then $A \leq_{\mathrm{P}} a^*b^*$.

f. If A is r.e., then $A \leq_{\mathrm{P}} A_{\mathrm{TM}}$.

Problem 2. Suppose you are given a polynomial time algorithm D (for "decision") that, on input of a boolean formula ϕ, decides if ϕ is satisfiable. Describe an efficient procedure S (for "search") that *finds* a satisfying assignment for ϕ. How many calls to D does S make?

Problem 3. Let $\mathit{MULT\text{-}SAT} = \{\langle\phi\rangle \mid \phi$ has at least ten satisfying assignments$\}$. Show that *MULT-SAT* is NP-complete.

Problem 4. A graph $G = (V, E)$ is said to be k-*colorable* if there is a way to paint its vertices using colors in $\{1, 2, \ldots, k\}$ such that no adjacent vertices are painted the same color. When k is a number, by k*COLOR* we denote the language of (encodings of) k-colorable graphs. The language *3COLOR* is NP-complete. (You can assume this.) Use this to prove that the language *4COLOR* is NP-complete, too.

Problem 5. *Problem 7.26 from your book, where a supporting picture can be found.* You are given a box and a collection of cards. Because of the pegs in the box and the notches in the cards, each card will fit in the box in either of two ways. Each cards contains two columns of holes, some of which may not be punched out. The puzzle is solved by placing all the cards in the box so as to completely cover the bottom of the box (ie, every hole position is blocked by at lesat one card that has no hole there). Let $\mathit{PUZZLE} = \{\langle c_1, \ldots, c_k\rangle \mid$ each c_i represents a card and this collection of cards *has* as solution$\}$. Show that *PUZZLE* is NP-complete.

Problem 6. Let

$$D = \{\langle p\rangle : \ p \text{ is a polynomial (in any number of variables) and } p \text{ has an integral root}\}.$$

Prove that D is NP-hard. Is it NP-complete?