Quiz 1

Your name:

Answer all questions. Please write neatly. Please be careful—know that little or no credit will be given for wrong answers. Questions may be worth varying numbers of points.

1. Circle the correct answer. No justification is required.

   (a) **True** or **False**: There is an efficient algorithm to decide if a multivariate polynomial over the integers has an integer root.

   (b) **True** or **False**: $L = \emptyset$ is a language.

   (c) **True** or **False**: All finite languages are DFA-acceptable.

   (d) **True** or **False**: An infinite language can have an infinite complement.

   (e) **True** or **False**: If A and B are regular then so is $\{xyz : x \in A, y \in B, z \in A\}$.

   (f) **True** or **False**: If there's a 10-state DFA that accepts L then there's a 20-state DFA that accepts L.

   (g) **True** or **False**: If M is an NFA then $(L(M))^*$ can be accepted by an NFA.

   (h) **True** or **False**: If M is an NFA and there is one abb-labeled path from the start state to a final state, and there is another abb-labeled path from the start state to a non-final state, then M neither accept nor rejects abb — the machine is *invalid*.

2. Complete the following sentence, being mathematically precise and following the conventions of your text: *A **DFA** is a five-tuple $M = (Q, \Sigma, \delta, q_0, F)$ where Q is a finite set, Σ is an alphabet, $q_0 \in Q$, $F \subseteq Q$, and δ is a function having domain* [_____] *and range* [_____].

3. List the first five strings, in lexicographic order, of $\{1, 01, 200\}^*$. Assume an ordering of characters $0 < 1 < 2$.

4. Draw a **DFA** for the language:

   $$L = \{x \in \{a, b, c\}^* : x \text{ contains at least one } a \text{ and at least one } b, \text{ but no } c\}$$

   Make your DFA use as few states as possible.

5. Carefully explain what it **means** if I say: "the DFA-acceptable languages are closed under intersection." Don't indicate if the statement is true or false—just provide a precise mathematical translation of the meaning of the claim.

That's all!