

## Problem Set 1 – Due Monday, September 29

*Recall that homeworks are due at 4:30 pm and are turned in at 2131 Kemper*

1. Show that  $n^2 + n$  is even for any integer  $n$ .
2. Prove that if  $n$  is an odd integer then there is an integer  $m$  such that  $n = 4m + 1$  or  $n = 4m + 3$ .
3. Suppose you draw  $n \geq 0$  distinct lines in the plane, one after another, none of the lines parallel to any other and no three lines intersecting at a common point. The plane will, as a result, be divided into how many different regions  $L_n$ ? Find an expression for  $L_n$  in terms of  $L_{n-1}$ , solve it explicitly, and indicate what is  $L_{10}$ .
4. How many  $n$ -disk legal configurations are there in the Tower of Hanoi problem? A “legal configuration” entails that no disk is larger than a disk beneath it on the same peg. All  $n$  disks have different diameters.
5. Prove that there exist irrational numbers  $a$  and  $b$  such that  $a^b$  is rational. (*Hint: try  $a = b = \sqrt{2}$* )