

Problem Set 2 – Due Monday, October 6, 2008

1. Translate the following sentences into a formula of sentential logic: “You must file form 1040 if you are single and have made over \$60,000, or if you are married and made over \$80,000, or if you itemize deductions. An exception is made for those whose tax home is outside of the United States and for those who have friends high up in the White House.”
2. Three students, A , B , and C , are suspected of cheating on an examination. When they are questioned by Judicial Affairs, they assert:

A : “ B copied and C is innocent”

B : “If A is guilty then so is C ”

C : “I am innocent”

Now answer the following questions:

- (a) If A spoke the truth and B lied, who is innocent and who copied?
 - (b) If everyone is innocent, who told the truth and who lied?
 - (c) If C lied and B told the truth, who is guilty?
3. Write out a logical expression, a truth table, and a Boolean circuit all of which realize the function “if s then p else q .”
 4. Let $\text{mux}(p_{00}, p_{01}, p_{10}, p_{11}, x_1, x_0) = p_{x_1 x_0}$. Write a boolean formula and then draw a circuit that computes mux . For the latter, use only **and**, **or**, and **not** gates.
 5. Prove that $\{\rightarrow, \neg\}$ is logically complete.
 6. Prove that $\{\wedge, \vee\}$ is not logically complete.