Problem Set 3 – Due Monday, October 13, 2008

1. (Velleman, p. 64, problems 8 and 9.) Complete the following table, answering whether the statement is true (True) or false (False) when the universe of discourse is as indicated.

	$\mathbb{R}$	$\mathbb{Z}$
$\forall x \exists y (2x - y = 0)$		
$\exists y \forall x (2x - y = 0)$		
$\forall x \exists y (x - 2y = 0)$		
$\forall x (x < 10 \rightarrow \forall y (y < x \rightarrow y < 9))$		
$\exists y \exists z (y + z = 100)$		
$\forall x \exists y (y > x \wedge \exists z (y + z = 100))$		

2. (Velleman, p. 72, problem 2.) Translate the negation of the following statements into formulas of quantification logic, introducing predicates as needed.

   (a) There is someone in the freshman class who doesn't have a roommate.

   (b) Everyone likes someone, but no one likes everyone.

   (c) $(\forall a \in A)(\exists b \in B)(a \in C \leftrightarrow b \in C)$

   (d) $(\forall y > 0)(\exists x)(ax^2 + bx + c = y)$

3. Translate the following into a formula of the predicate calculus. "A language L that is regular will have the following property: there will be some number N (that depends on L) such that if s is a string in L (a *string* is a sequence of characters) whose length is at least N then s can be written as xyz where y is not the empty string and xy^iz is in the language L for every nonnegative integer i." *Note 1: you do not have to understand this statement to do this problem. Note 2: you will see this specific statement again in ECS 120.*

4. A well-formed formula is said to be in *conjunctive normal form* (CNF) if it is the conjunct (and) of terms where each term is the disjunct (or) of variables or their complements. Convert the following formula into CNF: $\phi = A \wedge (B \rightarrow C)$. Can every formula be converted into a logically equivalent one in CNF? Explain your answer.

5. In class we described the problem TILING, where you are given a collection of t tile types, $1, \ldots, t$, and Boolean functions $S(k, k')$ and $T(k, k')$ with the following semantics: $S(k, k')$ is true iff a tile of type k may be put immediately to the left of a tile of type k', and $T(k, k')$ is true iff a tile of top k may be put immediately beneath a tile of type k'. Explicitly specify a collection of formulas Γ such that Γ is satisfiable (meaning that there is a truth assignment that satisfies every formula in Γ) iff the upper quadrant of the plane is tilable according to the S and T constraints.