# Quiz 1

1. Write down a truth table for the formula $\phi = P \wedge (P \rightarrow Q)$.

| $P$ | $Q$ | $\phi$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

2. Is it the case that $P \wedge Q \models\!\!\!=\!\!\!| P \wedge (P \rightarrow Q)$? *Yes.*

3. How many satisfying assignments does the formula $P \vee Q \vee R$ have? *7.*

4. Define what it means for a set of formulas $\Gamma$ to be *satisfiable* (do make sure that your quantifiers are clear): $\Gamma$ *is satisfiable if ... there exists a truth assignment $t$ that makes every formula $\phi \in \Gamma$ come out true.*

5. Let $T_n$ be the necessary and sufficient number of moves to solve the Towers of Hanoi problem using $n$ disks. Write an expression for $T_n$ ($n \geq 1$) in terms of $T_{n-1}$.

$$T_n = 2T_n + 1$$

6. Capture the logical content of the following sentence in a Boolean formula: *Nobody likes Mark except his roommates, who actually do like him.* Make your formula as succinct as possible. Use predicate symbols $L(x, y)$ (person $x$ likes person $y$), $R(x, y)$ (persons $x$ and $y$ are roommates), and the constant symbol $\mathsf{Mark}$. The universe of discourse is people.

   $(\forall x) L(x, \mathsf{Mark}) \leftrightarrow R(x, \mathsf{Mark})$

7. Is the following formula true or false when the universe of discourse is the set of real numbers?

   $(\forall x)(\forall y)(x < y \rightarrow (\exists z)(y - z = z - x))$. *True* (select $z$ as the midpoint between $x$ and $y$)

8. State "DeMorgan's law": $\neg(P \wedge Q) = \neg P \vee \neg Q$