Problem Set 1 – Due Wednesday, October 2

Recall from the course-information sheet that homeworks are due Wednesdays at 4:15 pm in 2131 Kemper, that you must acknowledge anyone with whom you collaborate, and that access to old solutions, of any sort, is strictly forbidden.

1. **(a)** A ball and a bat costs \$1.10 (total). The bat costs \$1 more than the ball. How much does the ball cost?

   (b) If your first impulse for (a) was wrong (admit it!), explain why you think that that was so.

   (c) I have a deck of cards that is supposed to respect the following rule: **if a card has an even number on one side, then it is red on the other.** I deal you four cards: **3, black, 8, red.** Which cards must you flip over to see if the rule is violated?

   (d) Some customers at a restaurant are supposed to respect the following law: **if someone is under 21 years of age, they're not allowed to drink alcohol.** Four people are drinking, and they-or-their-beverage are **23, tequila, 17, Pepsi**. Which people must we inspect the beverage of, and which beverage must we inspect the drinker of, to know if the drinking-age law was violated?

   (e) If problem (c) was less obvious than problem (d), explain why you think this is so.

2. Five misanthropes (all computer science professors) live on a triangular island of Australasia. The island's dimensions are 2 miles $\times$ 2 miles $\times$ 2 miles. Show that some two of the misanthropes must live within a mile of one another. (They won't be happy about it.) Hint: the *pigeonhole principle* says that if N items are placed into n boxes, where $n < N$, then some box must contain two or more items.

3. I am walking to my gate at the airport but need to pause for a few seconds to tie my shoes. In a few minutes I will get to a moving walkway. Normally I would walk while on it, but I could pause to tie my shoe while on the walkway, instead of doing so right now. Will I reach my gate faster, slower, or at the same time if I wait to tie my shoe until I'm on the moving walkway?

4. Show that $n^2 + n$ is even for any integer n.

5. Prove that if n is an odd integer then there is an integer m such that $n = 4m + 1$ or $n = 4m + 3$.

6. Suppose you draw $n \geq 0$ distinct lines in the plane, one after another, none of the lines parallel to any other and no three lines intersecting at a common point. The plane will, as a result, be divided into how many different regions L_n? Find an expression for L_n in terms of L_{n-1}, solve it explicitly, and indicate what is L_{10}.

7. How many n-disk legal configurations are there in the Tower of Hanoi problem? A "legal configuration" entails that no disk is larger than a disk beneath it on the same peg. All n disks have different diameters.

8. Prove that there exist irrational numbers a and b such that a^b is rational. *(Hint: try $a = b = \sqrt{2}$)*