Problem Set 2 – Due Wednesday, October 9

1. How many rising sequences are in the following sequence of numbers: $\pi = (1, 8, 3, 6, 4, 9, 7, 10, 5, 2)$? What are they? What is the least number of riffle shuffles that could have produced this ordering if the initial ordering of cards was $\pi_0 = (1, 2, 3, 4, 5, 6, 7, 8, 9, 10)$?

2. Write out a logical expression, a truth table, and a Boolean circuit all of which realize the function: if s then p else q.

3. Let $\mathsf{mux}(p_{11}, p_{10}, p_{01}, p_{00},\ x_1, x_0) = p_{x_1 x_0}$ (with all variables bits). Write a boolean formula, and then draw a circuit, that computes mux. For the latter, use only and, or, and not gates.

4. Translate the following sentences into a formula of sentential logic: "You must file form 1040 if you are single and have made over $60,000, or if you are married and made over $80,000, or if you itemize deductions. An exception is made for those whose tax home is outside of the United States and for those who have friends high up in the White House."

5. Three students, A, B, and C, are suspected of cheating on an examination. When they are questioned by SJA, they assert:

   A: "B copied and C is innocent"
   B: "If A is guilty then so is C"
   C: "I am innocent"

   Now answer the following questions:
   (a) If A spoke the truth and B lied, who is innocent and who copied?
   (b) If everyone is innocent, who told the truth and who lied?
   (c) If C lied and B told the truth, who is guilty?

6. Prove that $\{\rightarrow, \neg\}$ is logically complete.

7. Consider the parity function: $F_n(x_1, \ldots, x_n) = \oplus_{i=1}^{n} x_i$ where each x_i is boolean. Prove that, for every $n \geq 2$, there is no way to compute F_n using only AND and OR gates, and the constants 0 and 1.