SAPOR: Second-Order Arnoldi Method for Passive Order Reduction of RCS Circuits*

Yangfeng Su[1], Jian Wang[2], Xuan Zeng[2], Zhaojun Bai[3], Charles Chiang[4], Dian Zhou[2]

[1] Mathematics Dept., Fudan University, Shanghai 200433, P.R. China
[2] ASIC & System State-Key-Lab., Microelectronics Dept., Fudan University, Shanghai 200433, P.R. China
[3] Dept. of Computer Science, University of California, Davis, CA 95616, USA
[4] Advanced Technology Group, Synopsys Inc., Mountain View, CA 94043, USA

ABSTRACT

The recently-introduced susceptance element exhibits many prominent features in modeling the on-chip magnetic couplings. For an RCS circuit, it is better to be formulated as a second-order system. Therefore, corresponding MOR (model-order reduction) techniques for second-order systems are desired to efficiently deal with the ever-increasing circuit scale and to preserve essential model properties. In this paper, we first review the existing MOR methods for RCS circuits, such as ENOR and SMOR, and discuss several key issues related to numerical stability and accuracy of the methods. Then, a novel technique, SAPOR (**S**econd-order **A**rnoldi method for **P**assive **O**rder **R**eduction), is proposed to effectively address these issues. Based on an implementation of a generalized second-order Arnoldi method, SAPOR is numerically stable and efficient. Meanwhile, the reduced-order system also guarantees passivity.

1. INTRODUCTION

As the IC design and fabrication technology advances, interconnect delay becomes a dominant factor for determining the performance of the whole chip. This makes the accurate extraction and evaluation tools for on-chip interconnects an indispensable part of current VLSI EDA software. However, the ever-increasing signal frequency and circuit complexity together pose great challenges to interconnect modeling and simulation techniques.

As the operating frequency of the state-of-the-art IC chips continues to increase, it becomes a necessary practice to model the magnetic coupling effects of interconnects. The traditional 3-D inductance extraction methods are based on partial inductance concept, and the resulted inductance matrix is usually very large and dense [4], which may cause great difficulties to the subsequent simulation process. Recently, as an alterative approach, a new susceptance element has been introduced to model the on-chip magnetic couplings [2, 8]. The inherent characteristic of susceptance provides the advantage that as the distance increases, the mutual susceptance drops much faster than the mutual inductance does. As a result, the susceptance matrix is diagonally dominant, and can be sparsified by a simple truncation method without disrupting the positive definiteness. This enables the development of fast simulation methods.

Model-order reduction (MOR) techniques have been widely used to reduce the scale of the extracted interconnect circuits as well as to expedite the simulation. Usually, a linear circuit can be equivalently formulated in the form of a first-order system or a second-order system [7, 9]. Thus, MOR techniques can be classified into two categories accordingly.

For MOR of the first-order formulation, the pioneering work is AWE [6], which uses a reduced-order system to match the explicitly-calculated moments of the original system. However, AWE suffers from numerical instability and cannot generate high-order models. Therefore, some other MOR methods based on Krylov subspace techniques [3] were proposed. They often lead to a numerically stable order reduction process, which is highly desired for practical applications. Furthermore, special attention has been paid to maintain the passivity of the reduced-order model because common circuit simulation methods require provably passive models to ensure the stability of the simulation process. In [5], PRIMA was developed based on Arnoldi process, which may provide guaranteed passivity. However, PRIMA is not numerically efficient for large-scale circuits. More importantly, when directly applied to RCS circuits, PRIMA cannot guarantee passivity [9].

The techniques for MOR of the second-order formulation are developed in a similar manner. In [7], ENOR method was proposed for the second-order systems, which can generate a passive reduced-order model by utilizing the symmetry positive definite (s.p.d.) property of the system matrices. However, like AWE, ENOR uses a recursive formula to calculate the moments of the original system explicitly. ENOR is therefore not numerically stable. To address this issue, SMOR method was proposed in [9] trying to employ the Krylov subspace techniques. Based on a recursive relation similar to the one in ENOR, SMOR

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eliminates the auxiliary variables that are not orthonormalized in ENOR method, thus improves the numerical stability and accuracy. However, as demonstrated later in this paper, the projection subspace formed by SMOR method is only an approximation of the space spanned by the moments of the original system. As a result, the reduced-order system by SMOR cannot match the moments of the original system exactly. Therefore, the accuracy of the reduced-order model cannot be guaranteed.

It is better to formulate an RCS circuit as a second-order system since many good properties of the susceptance matrix can be preserved in this form [9]. However, up to now, existing MOR techniques for the second-order formulation still leave many key issues to be resolved, such as numerical stability and accurate moment matching. A Krylov subspace based MOR technique is much more attractive for the second-order formulation, just like PRIMA for the first-order systems.

Recently, Bai and Su proposed a second-order Arnoldi method (SOAR) [1] for solving the quadratic eigenvalue problem. In this paper, we present a modified version of this method for the model-order reduction of second-order formulation of RCS circuits. By fully utilizing the symmetric positive definiteness of the system matrices, the proposed novel method, SAPOR (**S**econd-order **A**rnoldi method for **P**assive **O**rder **R**eduction), generates guaranteed passive models. Furthermore, since SAPOR is a numerical procedure similar to the well-known Arnoldi procedure, it is numerically more stable and efficient than ENOR. Also, the reduced-order model can accurately match the moments of the original system, which outperforms SMOR.

The rest of the paper is organized as follows. In Section 2, we review the formulation of RCS circuits and outline ENOR and SMOR methods. Our new SAPOR approach, a modified second-order Arnoldi-based order reduction process, is described in Section 3. Several numerical examples are reported in Section 4 to demonstrate the effectiveness of our proposed method. Concluding remarks are given in Section 5.

2. BACKGROUND REVIEW

2.1 MOR of RCS Circuit

Since the susceptance matrix can be regarded as the inverse of the inductance matrix, the nodal equations for an RCS circuit may be formulated as

$$\begin{bmatrix} G & E_S \\ -SE_S^T & 0 \end{bmatrix}\begin{bmatrix} V(t) \\ I_S(t) \end{bmatrix} + \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} \dot{V}(t) \\ \dot{I}_S(t) \end{bmatrix} = \begin{bmatrix} BJ(t) \\ 0 \end{bmatrix} \tag{1}$$

where $V(t)$ and $I_S(t)$ are the unknown vectors of node voltage and susceptance current; $J(t)$ is the current source vector; G, C and S are matrices of conductance, capacitance and susceptance, respectively; E_S and B are incidence matrices for susceptances and current sources. Performing the Laplace transform on (1), we have the nodal equation in frequency domain as

$$\left(\begin{bmatrix} G & E_S \\ -SE_S^T & 0 \end{bmatrix} + s\begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix}\right)\begin{bmatrix} V(s) \\ I_S(s) \end{bmatrix} = \begin{bmatrix} BJ(s) \\ 0 \end{bmatrix} \tag{2}$$

where $V(s)$, $I_S(s)$ and $J(s)$ are Laplace transforms of $V(t)$, $I_S(t)$ and $J(t)$, respectively. Obviously, this is a first-order system in terms of s.

In most applications, only nodal voltages are of interests, and the susceptance currents in vector I_S are intermediate variables. Therefore, we may eliminate I_S from (2). From the lower part of the above frequency domain equation (2), it can be obtained that

$$I(s) = \frac{1}{s} SE_S^T V(s)\,. \tag{3}$$

Substituting (3) into the upper part of (2), we have

$$\left(sC + G + \frac{1}{s}\Gamma\right)V(s) = BJ(s) \tag{4}$$

where $\Gamma = E_S S E_S^T$. This second-order formulation is equivalent to (2), except for the eliminated susceptance currents.

For a circuit with N nodes (excluding the ground node), the order of equation (4) is also N. If it is desired to reduce the circuit to a lower order n, a MOR technique can be employed to construct an orthonormal basis Q for the moment vectors of nodal voltage up to order n. Then by performing an orthogonal projection on the original system using Q, we obtain a reduced-order system of the same form

$$\left(s\widetilde{C} + \widetilde{G} + \frac{1}{s}\widetilde{\Gamma}\right)\widetilde{V}(s) = \widetilde{B}J(s) \tag{5}$$

where $\widetilde{C} = Q^T C Q$, $\widetilde{G} = Q^T G Q$, $\widetilde{\Gamma} = Q^T \Gamma Q$, $\widetilde{V} = Q^T V$, and $\widetilde{B} = Q^T B$.

For an RCS circuit in form of (4), the matrices C, G and Γ are all symmetry positive semi-definite. In [7], it is proved that the orthogonal projection preserves the passivity of the original system. This means the reduced-order system (5) has guaranteed passivity.

2.2 ENOR Method

In ENOR method [7], the moments of the original system are explicitly generated and orthonormalized via a recursive formula. By substituting variable s with $s = s_0(1-z)$, where s_0 is a selected frequency expansion point, and introducing an auxiliary quantity $Y(z) = \frac{V(z)}{1-z}$, we have the following recursive relation (for $k \geq 0$):

$$PV_k = s_0 C V_{k-1} - \frac{1}{s_0}\Gamma Y_{k-1} + BJ_k \tag{6}$$

$$Y_k = V_k + Y_{k-1}, \quad V_{-1} = Y_{-1} = 0 \tag{7}$$

where V_k, Y_k and J_k are actually the k-th moments of V, Y and J, expanded about frequency s_0; and $P = s_0 C + G + \frac{1}{s_0}\Gamma$. Then, the moments of V can be generated by this recursive scheme and be orthonormalized by a Gram-Schmidt process, up to any desired order.

However, the moments computed explicitly by the above recursive relation are prone to numerical instability. Moreover, only the V vectors are orthonormalized in the Gram-Schmidt process, and Y vectors are calculated accordingly without orthonormalization. As noted in [9], when the iteration goes up, the magnitude of Y vectors could grow rapidly, which may also cause numerical instability. For these reasons, ENOR method is not numerically stable.

2.3 SMOR Method

To address the numerical stability issue, SMOR [9] uses a method based on Krylov subspace to construct an orthonormal basis for model-order reduction.

From (7), it can be derived that

$$Y_k = \sum_{i=-1}^{k} V_k \text{ , for } k \geq -1 . \tag{8}$$

Since impulse responses are normally required for moment matching, in the rest of the paper, we assume that $J_k = 0$ for $k \geq 1$. Substituting (8) into (6), we may have a new recurrence relation

$$PV_k = s_0 CV_{k-1} - \frac{1}{s_0}\Gamma \sum_{i=-1}^{k-1} V_i \text{ , for } k \geq 1 \tag{9}$$

$$V_{-1} = 0, \quad V_0 = P^{-1} BJ_0 . \tag{10}$$

Thus the Y vectors are eliminated.

Next, the SMOR method constructs the generalized Krylov subspace according to the above formula. To speed up the model-order reduction process and to avoid error accumulation, SMOR only preserves the first three terms on the RHS of (9) and uses the following recurrence

$$PV_k' = s_0 CV_{k-1}' - \frac{1}{s_0}\Gamma V_{k-1}' - \frac{1}{s_0}\Gamma V_{k-2}' \text{, for } k \geq 1 \tag{11}$$

where V_k' is an approximation of V_k, as claimed in [8]. Then using the simplified recursive relation (10) and (11), V_k' is generated and orthonormalized.

By eliminating Y vectors in the recursive formula, SMOR method improves the numerical stability of the MOR process. However, because of the simplification made in (11), the subspace formed by SMOR method is only an approximation of the space spanned by the moments of the original system. Therefore, the reduced-order system by SMOR cannot match the moments of the original system exactly. As a result, the accuracy of the reduced-order model cannot be guaranteed.

3. SAPOR METHOD

As mentioned in Section 2.1, in order to construct a stable and passive order reduction process for RCS circuits, we should construct an orthonormal basis of the space spanned by the moments of V and use it as the projection matrix Q_n. Then, by performing an orthogonal projection on the original system (4), we can obtain the reduced-order system as in (5).

Under the assumption that impulse response is required, i.e. $J_k = 0$ for $k \geq 1$, the nodal equation (4) of an RCS circuit can be rewritten as

$$\left(s^2 C + sG + \Gamma\right) V(s) = sBJ_0 . \tag{12}$$

Shifting it with $s = s_0 + \sigma$, we have

$$\left(\sigma^2 C + \sigma D + K\right) V(\sigma) = b_0 + b_1 \sigma \tag{13}$$

where $D = 2s_0 C + G$, $K = s_0^2 C + s_0 G + \Gamma$, $b_0 = s_0 BJ_0$, and $b_1 = BJ_0$.

Applying Taylor expansion to V around expansion point s_0, we have

$$\left(\sigma^2 C + \sigma D + K\right) \cdot \left(V_0 + V_1 \sigma + V_2 \sigma^2 + \cdots\right) = b_0 + b_1 \sigma \tag{14}$$

where V_0, V_1, V_2, … are the moments of V. By comparing two sides of the above equation, we may have the following recurrence relation

$$V_0 = K^{-1} b_0 \tag{15}$$

$$V_1 = K^{-1}\left(-DV_0 + b_1\right) \tag{16}$$

$$V_j = K^{-1}\left(-DV_{j-1} - CV_{j-2}\right), \; j = 2, 3, 4, \cdots . \tag{17}$$

However, it would be numerically unstable if one used the above recurrence to explicitly calculate the moments of V. Instead, a Krylov subspace based technique is more desirable. Equation (13) is in the form of a quadratic parameterized matrix equation (QPE). And in [1], a second-order Arnoldi method (SOAR) was proposed for the quadratic eigenvalue problem, which can be simply regarded as a QPE with zero RHS. In this section, we will generalize the SOAR method and apply it for the MOR of RCS circuits.

3.1 System Linearization

Introduce a new variable $Z(\sigma)$ satisfying

$$\sigma CV(\sigma) + Z(\sigma) = b_1 . \tag{18}$$

Substituting (18) into (13), we have

$$-\sigma Z + \sigma DV + KV = b_0 . \tag{19}$$

Combining (18) and (19), we get

$$\left(I - \sigma A\right)\begin{bmatrix} V \\ Z \end{bmatrix} = \begin{bmatrix} q_0 \\ p_0 \end{bmatrix} \tag{20}$$

where $A = \begin{bmatrix} -K^{-1}D & K^{-1} \\ -C & 0 \end{bmatrix}$; $q_0 = K^{-1} b_0$ and $p_0 = b_1$.

By moving $\left(I - \sigma A\right)$ to the RHS of (20) and performing a Maclaurin series expansion, we have

$$\begin{bmatrix} V \\ Z \end{bmatrix} = \left(I + \sigma A + \sigma^2 A^2 + \sigma^3 A^3 + \cdots\right)\begin{bmatrix} q_0 \\ p_0 \end{bmatrix}. \tag{21}$$

Obviously, $A^{i-1}\begin{bmatrix} q_0 \\ p_0 \end{bmatrix}$ is the i-th moment of $\begin{bmatrix} V \\ Z \end{bmatrix}$, q_0 and p_0 are actually the first moments of V and Z, respectively.

Equation (20) is a linearized form of (13) with RHS independent of σ. If $\begin{bmatrix} V \\ Z \end{bmatrix}$ is the solution of (20), V must be the solution of (13), which means that the upper part of the i-th moment of $\begin{bmatrix} V \\ Z \end{bmatrix}$, i.e. $\begin{bmatrix} I & 0 \end{bmatrix} A^{i-1}\begin{bmatrix} q_0 \\ p_0 \end{bmatrix}$, must be equal to the i-th moment of V.

Algorithm: Generalized SOAR
Input: A, q_0, p_0 and an integer order n
Output: the orthonormal matrix Q_n

1. $\beta = \|q_0\|$
2. $\begin{bmatrix} q_1 \\ p_1 \end{bmatrix} = \frac{1}{\beta}\begin{bmatrix} q_0 \\ p_0 \end{bmatrix}$
3. For $j = 1, 2, \cdots, n-1$
4. $\quad\begin{bmatrix} q_{j+1} \\ p_{j+1} \end{bmatrix} = A\begin{bmatrix} q_j \\ p_j \end{bmatrix}$
5. $\quad$For $i = 1, 2, \cdots, j$
6. $\quad\quad h_{ij} = q_i^T q_{j+1}$
7. $\quad\quad\begin{bmatrix} q_{j+1} \\ p_{j+1} \end{bmatrix} = \begin{bmatrix} q_{j+1} \\ p_{j+1} \end{bmatrix} - h_{ij}\begin{bmatrix} q_j \\ p_j \end{bmatrix}$
8. $\quad$End
9. $\quad h_{j+1\ j} = \|q_{j+1}\|$
10. $\quad$If $h_{j+1\ j} \cong 0$, stop (breakdown)
11. $\quad\begin{bmatrix} q_{j+1} \\ p_{j+1} \end{bmatrix} = \frac{1}{h_{j+1\ j}}\begin{bmatrix} q_{j+1} \\ p_{j+1} \end{bmatrix}$
12. End
13. $Q_n = [q_1 \ \cdots \ q_n]$

Figure 1. Generalized SOAR algorithm

3.2 Orthonormalization Process

In this subsection, we will use a generalized version of the second-order Arnoldi method (SOAR) [1] to construct an orthonormal basis of the space spanned by the moments of V.

The generalized version of SOAR is described in Figure 1. This variant of SOAR is similar as the original algorithm in [1] except that in the original algorithm, the initial vector p_0 is always set to an all-zero vector.

To ensure the integrity and concision, we have the following theorems:

Theorem 1: If the generalized SOAR algorithm does not stop in step 10, vectors $q_1, q_2, \ldots, q_n$ form an orthonormal basis of the moment space of V.

Theorem 2: If we use the matrix Q_n obtained by the generalized SOAR algorithm to perform a projection on the system (4) and obtain the projected system (5), the moments of the reduced-order system (5) will match the first n moments of the original system (4).

The proof of Theorem 1 is similar to the one for the original SOAR algorithm [1]. Theorem 2 can be proved subsequently. The proofs for these theorems are omitted here due to the limitation of space. Theorem 1 ensures that Q_n is an orthonormal matrix. Therefore, after a congruent transformation using Q_n, the obtained system (5) may preserve the passivity of the original system (4). Theorem 2 guarantees that the reduced-order system can accurately match the moments of the original system.

Similar technology as described in [1] can be employed when the algorithm stops at step 10. The techniques for tackling with this breakdown case can also help to improve the stability of the orthonormalization process [1].

In the orthonormalization process, the only matrix inversion required is K^{-1}. Since K is both symmetric positive definite (s.p.d.) and sparse, there are many efficient methods to compute a sparse Cholesky factorization of K, and then efficiently apply it to the matrix-vector multiplication involving K^{-1}. Moreover, the generalized SOAR can be modified to a memory-saving variant, where the vectors $p_1, p_2, \ldots, p_n$ are not explicitly saved. Unfortunately, due the limited length of the paper, it cannot be expanded here.

3.3 Summary of SAPOR

Finally, our novel technique SAPOR (**S**econd-order **A**rnoldi method for **P**assive **O**rder **R**eduction), can be outlined as follows:

1) Formulating the RCS circuit as the second-order system in (4);
2) Shifting (4) with $s = s_0 + \sigma$ to obtain (13);
3) Introducing a variable $Z(\sigma)$ satisfying (18), and linearizing the system to the form of (20);
4) Using the generalized SOAR algorithm in Figure 1 to construct the orthonormal matrix Q_n;
5) Performing an orthogonal projection on the original system, and obtaining the reduced-order system as in (5).

SAPOR provides the following advantages over the previous RCS circuit MOR techniques, such as ENOR and SMOR.

First, in ENOR method, the moments of the voltage vector are explicitly calculated and orthonormalized, whereas SAPOR utilizes the Krylov subspace technique to achieve a numerically more stable orthonormalization process. For the MOR of second-order formulation, the relationship between SAPOR and ENOR is just like that between Arnoldi-based method and AWE for the first-order system. Our experiments show that SAPOR is numerically far more stable than ENOR, and is quite preferable for practical applications.

Second, as demonstrated in Theorem 1, SAPOR can correctly generate the subspace spanned by the moments of nodal voltage vector. Theorem 2 further guarantees that the reduced-order system can accurately match the moments of the original system so that SAPOR outperforms SMOR.

Last but not least, by comparing the processes of SAPOR, ENOR and SMOR, we can find that the three algorithms employ similar operations in the MOR process up to the same order. This means the three methods have similar time complexity.

4. NUMERICAL EXPERIMENTS

In this section, we present several numerical experiments to demonstrate the efficiency of the proposed SAPOR method, and compare SAPOR with ENOR and SMOR methods. All three methods are implemented in MATLAB. Experiments are run on a PC with Intel Pentium IV 1.7G CPU and 1G RAM.

An interconnect circuit from industry serves as the example. The circuit, as shown in Figure 2, consists of an 8-bit bus and two shielding lines (grey ones in the figure). It should be noted that there are capacitive and magnetic couplings between any two of these 10 lines, which are not shown in the figure for simplicity and clarity reasons. The near end of the first line is driven by a current source as the excitation. The voltage of node ***A***, which is at the far end of the same line, is of our interests. We use a susceptance-based extraction tool to acquire the circuit topology and element parameters. The nodal equation of the obtained RCS circuit, in the form of (1), has 330 nodal voltages and 160 susceptance currents; hence there are total 490 unknown variables. This means the original system has an order of 490.

We may use the frequency response at node ***A*** as a criterion for judging the accuracy of the reduced-order system. We begin by using SAPOR to reduce the original system to different orders and compare the accuracy of the reduced-order systems. In Figure 3, the frequency responses are plotted for the original system and the reduced-order systems with orders 40, 60 and 80. Errors of the reduced-order systems are given in Figure 4. At the low frequency region, i.e., the region up to 2 GHz, all three systems can match the exact response perfectly with the error magnitude around 10^{-12}, which is mainly caused by the finite word length of the computer. As the order goes up, the reduced system can match the exact response in a wider frequency range due to more matched-moments. This is important in the simulation of high-speed circuits because wider matched frequency range means the reduced system can accommodate signals with faster transition time.

For the sake of comparison, we perform ENOR and SMOR to the same circuit. The errors of the reduced-order systems obtained by ENOR and SMOR methods are plotted in Figure 5 and Figure 6, respectively. For the ENOR method, we can find that as the reduced-order increases, the error almost stays at the same magnitude. This may well exhibit the numerical instability of ENOR for high-order models. As to the SMOR method, it has a higher accuracy compared with ENOR. However, as aforementioned, SMOR cannot exactly match the moments of the reduced-order system with the moments of the original system. Due to this systematical error, SMOR never reaches the accuracy as high as SAPOR does for this example.

Finally, we give one figure to depict the errors of these three MOR techniques with the same reduced order. We set the order of the reduced system to be 80 for all three methods and we obtain the frequency responses of node ***A*** for the three reduced-order systems. Plot in Figure 7 is the error comparison for these responses. As we can see, ENOR is most inaccurate among the three methods; this is due to the numerical instability of the recursive reduction process in explicit moment computation. With improved orthonormalization algorithm, SMOR is more accurate than ENOR. SAPOR performs the best, with the response almost indistinguishable from the exact one at more than 15 GHz. This is because that SAPOR can accurately generate the subspace spanned by the moments of the original system. As to other reduction orders, the comparison results are similar, thus are neglected here due to the space limitation.

For more complicated circuits, which require the reduced-order systems to have higher orders, more significant accuracy improvements by SAPOR can be expected. This means that the advantages of SAPOR will be more prominent as the circuit scale continuously increases.

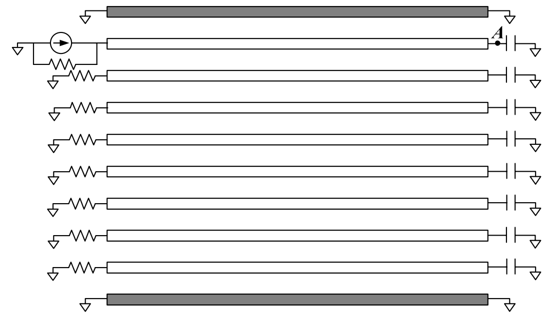

Figure 2. 8-bit bus with two shielding lines

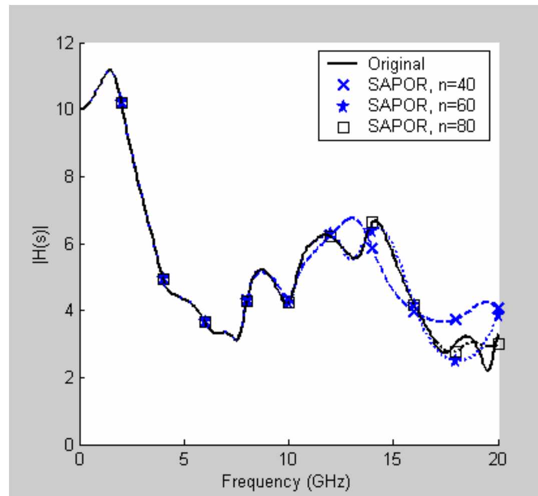

Figure 3. Comparison of frequency responses by SAPOR with three different orders

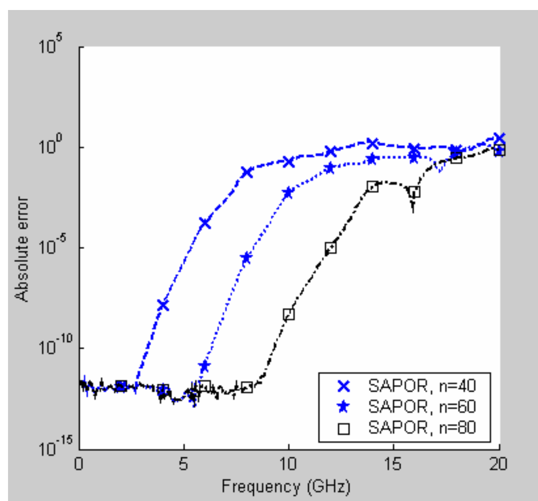

Figure 4. Errors of SAPOR with three different orders

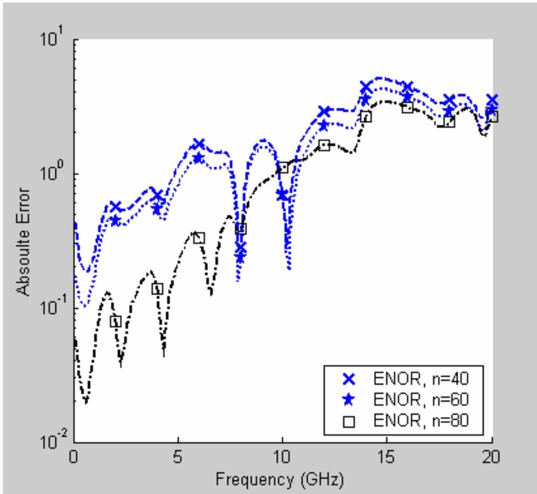

Figure 5. Errors of ENOR with three different orders

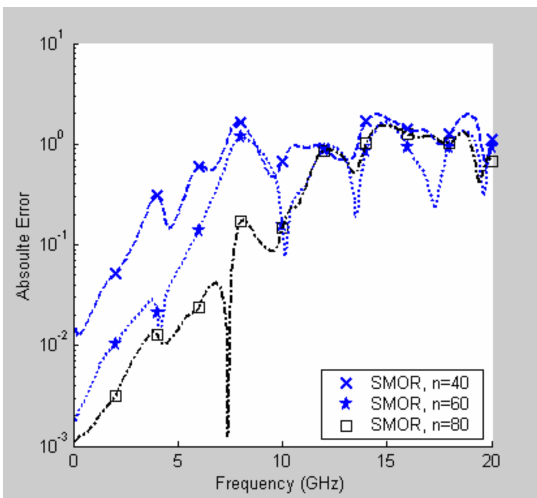

Figure 6. Errors of SMOR with three different orders

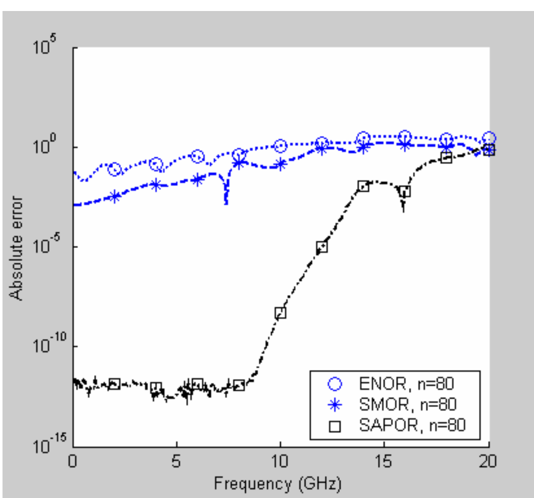

Figure 7. Errors of ENOR, SMOR and SAPOR with the same reduced-order

5. CONCLUSIONS

In this paper, a novel technique, SAPOR (**S**econd-order **A**rnoldi method for **P**assive **O**rder **R**eduction), is proposed for the model-order reduction of RCS circuits. By exploiting the symmetric positive definiteness of the system matrices, SAPOR can generate guaranteed passive models for RCS circuits, which is highly desired in the simulation with nonlinear elements. Based on the generalized second-order Arnoldi method (SOAR), the new model-order reduction process is both numerically stable and efficient. The resulted reduced-order model can accurately match the moments of the original system. For larger and more complex circuits, more significant improvements by SAPOR are expected.

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