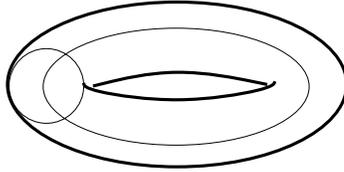


Handout - ECS175 Finding normal vectors on a torus

We develop a parametric representation of the torus by thinking of it as the surface swept out by a little circle rotating around on a big circle. Every point on the surface can be parameterized by an angle i on the little circle and an angle j on the big circle.



We let r_l be the radius of the little circle and r_b the radius of the big circle. The parametric description of a surface point $p(i, j)$ is:

$$[X(i, j), Y(i, j), Z(i, j)] = [(r_b + r_l \cos i) \cos j, (r_b + r_l \cos i) \sin j, r_l \sin i]$$

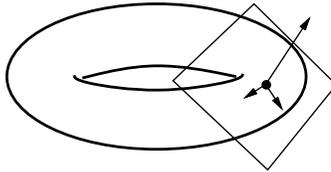
Lets introduce a specific example. Say $r_b = 5$ and $r_l = 1$. Then

$$p(i, j) = [(5 + \cos i) \cos j, (5 + \cos i) \sin j, \sin i]$$

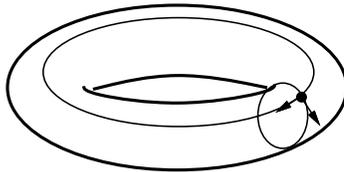
The point on the surface corresponding to the parameter values $i = \pi/3 = 60$ degrees and $j = \pi/2 = 45$ degrees is:

$$p(\pi/3, \pi/2) = [(5 + 1/2)/\sqrt{2}, (5 + 1/2)/\sqrt{2}, \sqrt{3}/2]$$

One approach to finding the normal vector at $p(i, j)$ is to first find two tangent vectors at $p(i, j)$. Any two tangent vectors lie in the tangent plane at $p(i, j)$, and the normal at $p(i, j)$ is the normal to the tangent plane at $p(i, j)$. So we can find the normal vector by taking the cross product of any two tangent vectors.



Two tangent vectors that are easy to find are the vectors tangent to the curve on the torus formed by keeping the value of i at $p(i, j)$ fixed, and varying j , and the curve formed by keeping the value of j fixed and varying i . The parametric equations for these two curves are formed by just substituting in the fixed values of i or j into the parametric description of the surface.



So at point $p(\pi/3, \pi/4)$ on our torus, we get the curves:

$$c_i(j) = p(\pi/3, j) = [(5 + 1/2) \cos j, (5 + 1/2) \sin j, \sqrt{3}/2]$$

$$c_j(i) = p(i, \pi/2) = [(5 + \cos i)/\sqrt{2}, (5 + \cos i)/\sqrt{2}, \sin i]$$

To find tangent vectors to those curves, we first take the derivative with respect to the parameter (j for the first one, i for the second). Remember the derivative of \sin is \cos , and the derivative of \cos is $-\sin$.

$$\delta(c_i)/\delta j = [(5 + 1/2) - \sin j, (5 + 1/2) \cos j, 0] = [(r_b + r_l \cos i) - \sin j, (r_b + r_l \cos i) \cos j, 0]$$

$$\delta(c_j)/\delta i = [(-\sin i)/\sqrt{2}, (-\sin i)/\sqrt{2}, \cos i] = [r_l(-\sin i) \cos j, r_l(-\sin i) \sin j, r_l \cos i]$$

We can simplify these a little, since we can normalize them to whatever length we like; we're just concerned with the direction.

$$\delta(c_i)/\delta j = [-\sin j, \cos j, 0]$$

$$\delta(c_j)/\delta i = [-\sin i \cos j, -\sin i \sin j, \cos i]$$

Evaluating those derivatives at $p(i, j)$ gives us two tangent vectors at that point:

$$t_i = c_i(\pi/4) = [\sqrt{2}, \sqrt{2}, 0]$$

$$t_j = c_j(\pi/3) = [\sqrt{3}/(2\sqrt{2}), \sqrt{3}/(2\sqrt{2}), 1/2]$$

Finally, to find the normal vector, we compute $t_i \times t_j$. It's a good idea to normalize the normal vector afterwards.

This is actually a lot easier to let the computer figure out than it is to do yourself on paper. Here's the piece of code from the project that computes the normal vectors for the torus:

```

/* vertex position */
vx = cos(jangle)*(rout+cos(iangle)*rin);
vy = sin(jangle)*(rout+cos(iangle)*rin);
vz = sin(iangle)*rin;
/* tangent vector with respect to big circle */
tx = -sin(jangle);
ty = cos(jangle);
tz = 0;
/* tangent vector with respect to little circle */
sx = cos(jangle)*(-sin(iangle));
sy = sin(jangle)*(-sin(iangle));
sz = cos(iangle);
/* normal is cross-product of tangents */
nx = ty*sz - tz*sy;
ny = tz*sx - tx*sz;
nz = tx*sy - ty*sx;
/* normalize normal */
length = sqrt(nx*nx + ny*ny + nz*nz);
nx /= length;
ny /= length;
nz /= length;

```