ECS 122A: Algorithm Design and Analysis — Handout ??
UC Davis — Charles Martel — April 1, 2003

Problem Set 0—NOT TO BE HANDED IN

This problem set is to check your knowledge of some basic facts used in this course. Every question listed here should be solvable for you based on what you've learned in the prerequisites for 122A (i.e. ECS 20 or 100, and ECS 110). Everything here is reviewed in the first few chapters of the 122A textbook or the appendix, so look through that material if you need help with anything. Feel free to see me or the TA if you have troubles with anything here, and you can get help at the first discussion.

Important Note: This is just a self test. It is not to be turned in or graded. Also, this problem set does not cover all the background you are expected to have, just some of the more fundamental topics! Solutions will be posted Friday.

1. Compute each of the following:

$$\lfloor 10.9 \rfloor = \qquad\qquad \lceil 10.9 \rceil =$$
$$|-1.1| = \qquad\qquad -|1.1| =$$

2. Compute the following (here we use the notation of the textbook: "log" means "log base 10," "ln" means "log base e," and "lg" means "log base 2"):

$$\log(10000) = \qquad\qquad \ln e^{2x} =$$
$$(\ln x)/(\ln 2) = \qquad\qquad (\lg x^n)/(n \lg x) =$$

3. In ANSI C, there is no lg() function. Instead we have only a ln() function (again, this is the natural logarithm) which is called `log()`. Assuming x is some positive real, say you wanted to compute $\lg(x)$. Describe how you would do this using the `log()` function. You may use any standard C functions in your solution.

4. Let n be a positive integer. We invoke the following code; how many times (as a function of n) is line 2 executed before the loop terminates? (Hint: Try some examples to help figure this out. Start with n being a power of 2, then generalize.)

   1: **while** $n > 0$ **do**
   2: $\quad n \leftarrow \lfloor n/2 \rfloor$
   3: **end while**

5. Compute each of the following:

$$\sum_{i=1000}^{2000} i = \qquad \sum_{k=0}^{\infty} 3^{-k} =$$

$$\lg\left(\prod_{k=1}^{n} 2^k\right) = \qquad \sum_{i=1}^{m}\sum_{k=1}^{n} ix =$$

6. Which statements below are true?:

   (a) $42n^2 \in O(41n^2)$ $\quad$ $(f)\log n \in O(\log\log n)$

   (b) $3^n \in O(2^n)$

   (c) $\lg n \in O(\lg^2 n)$

   (d) $100 \in O(10)$

   (e) $\sqrt{n} \in O(\lg n)$

7. You are given two fair dice; one is green and one is red. Each die has the numbers 1 through 6 on it. Let G be a random variable representing the value of the green die after a roll, and let R be the corresponding random variable for the red die. Compute the following:

$$\Pr[G = 1 \ \wedge \ R = 1] = \qquad \Pr[G + R = 6] =$$

8. Give the big O run time for a good algorithm to solve each problem, assume there are n data items if not indicated:

   (a) Finding a key in a balanced binary search tree

   (b) Adding an item to a balanced binary search tree

   (c) Merging two sorted lists each with $n/2$ items

   (d) Sorting a list

   (e) Adding a new element to a sorted array

   (f) Testing if an undirected graph with n vertices and m edges is connected

   (g) Adding a new element to a queue

   (h) Adding a new element to a stack