0-1 Knapsack problem

Problem statement:
- Given $n$ items $\{1, 2, \ldots, n\}$
- Item $i$ is worth $v_i$, and weight $w_i$
- Find a most valuable subset of items with total weight $\leq W$

Problem statement, mathematically:

Find a subset $S \subseteq \{1, 2, \ldots, n\}$ such that

\[
\text{maximize } \sum_{i \in S} v_i \\
\text{subject to } \sum_{i \in S} w_i \leq W
\]

Rule: have to either take an item or not take it – can’t take part of it.
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Three possible greedy strategies:

1. Greedy by highest value $v_i$

2. Greedy by least weight $w_i$

3. Greedy by largest value density $\frac{v_i}{w_i}$

*All three approaches generate feasible solutions. However, we cannot guarantee that any of them will always generate an optimal solution!*
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Example:

<table>
<thead>
<tr>
<th></th>
<th>$v_i$</th>
<th>$w_i$</th>
<th>$v_i/w_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Total weight $W = 5$

Greedy by value density $v_i/w_i$:
- take items 1 and 2.
- value = 16, weight = 3
- Leftover capacity = 2

Optimal solution:
- take items 2 and 3.
- value = 22, weight = 5
- no leftover capacity

Question: how about greedy by highest value? by least weight?