1. Let $T(n)$ be defined by the following recurrence relation

$$\begin{cases} T(0) = T(1) = 1 \\ T(n) = T(n-1) + T(n-2) + 1 \quad \text{for} \quad n \geq 2 \end{cases}$$

   Show that

$$T(n) = 2F_n - 1 \quad \text{for} \quad n \geq 0,$$

   where F_n is the nth Fibonacci number, i.e.,

$$\begin{cases} F_0 = F_1 = 1; \\ F_n = F_{n-1} + F_{n-2} \quad \text{for} \quad n \geq 2. \end{cases}$$

2. Find the solution of the recurrence relation $f_n = f_{n-1} + f_{n-2}$ with $f_0 = f_1 = 1$.

3. Show by mathematical induction that $T(n) = \lg n + 1$ is the solution of the recurrence relation

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ T(\frac{n}{2}) + 1 & \text{otherwise} \end{cases}$$

   Assume that $n = 2^k$, $k \geq 0$.