1. (a) Prove that \((n + 3)^3 = \Theta(n^3)\)
(b) Prove that for any real constants \(a\) and \(b\), where \(b > 0\),
\[(n + a)^b = \Theta(n^b)\]

Note: to establish the relationship \(f(n) = \Theta(g(n))\), we need to find the proper constants \(c_1\), \(c_2\) and \(n_0\) such that \(0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)\) whenever \(n \geq n_0\).

2. (a) Is \(2^{n+1} = O(2^n)\)? why? (b) Is \(2^m = O(2^n)\)? why?

3. Order the following functions into a list such that if \(f(n)\) comes before \(g(n)\) in the list then \(f(n) = O(g(n))\). If any two (or more) of the same asymptotic order, indicate which.
(a) Start with these basic functions
\[n, 2^n, n \lg n, n^3, \lg n, n - n^3 + 7n^5, n^2 + \lg n\]
(b) Combine the following functions into your answer for part (a). Assume that \(0 < \epsilon < 1\).
\[e^n, \sqrt{n}, 2^{n-1}, \lg \lg n, (\sqrt{2})^{\lg n}, \ln n, (\lg n)^2, n!, n^{1+\epsilon}, 1\]

4. Use the master theorem to give tight asymptotic bounds for the following recurrences.
(a) \(T(n) = 2T(n/4) + 1\)
(b) \(T(n) = 2T(n/4) + \sqrt{n}\)
(c) \(T(n) = 2T(n/4) + n\)
(d) \(T(n) = 2T(n/4) + n^2\)

5. Give asymptotic upper and lower bounds for \(T(n)\) in each of the following recurrences. Assume that \(T(n)\) is constant for sufficient small \(n\), and \(c\) is a constant. Make your bounds as tight as possible, and justify your answers.
(a) \(T(n) = T(n - 1) + 1/n\)
(b) \(T(n) = T(n - 1) + c^n\), where \(c > 1\) is some constant
(c) \(T(n) = 2T(n - 1) + 1\)
(d) \(T(n) = 2T(\frac{n}{2}) + \sqrt{n}\)
(e) \(T(n) = 3T(\frac{n}{3}) + cn\)
(f) \(T(n) = 27T(\frac{n}{3}) + cn^3\)
(g) \(T(n) = 5T(\frac{n}{3}) + cn^2\)