1. Summary (abstract)

What is the mathematical problem of this report about? What is your solution? What is your finding?

2. Introduction of the main problem

Statement of the problem: the linear least squares problem \( \min_\beta \| X\beta - b \| \)

(Background) When \( X \) is nonsingular, .... normal equation and QR decomposition

(Purpose of this project) We study the solution of the LS when when \( X \) is singular.

3. Definitions (tools/theory) needed:

rank deficient, singular value decomposition, pseudo-inverse

4. Algorithms

(a) \( \beta = X \backslash y \) (why not use)

(b) \( \beta = \text{pinv}(X, \text{tol})y \) with different drop tolerance values \( \text{tol} \) \( \text{pinv}(X) \) uses the default tolerance value.

(c) \( \beta = V\Sigma^+U^Ty \), where \( X = U\Sigma V^T \) is the SVD of \( X \), \( \Sigma^+ \) is defined with with a drop tolerance value \( \text{tol} \)

Note that Algorithms (b) and (c) are essentially the same.

5. Numerical examples

Use the Problem 5.6 and shaw.m to illustrate the key finding:

The accuracy (measured in the relative error) of the computed solution strongly depends on the drop tolerance value “\( \text{tol} \)” in \( \text{pinv} \) (or say SVD).

Here are numerical results in plots for the Shaw problem to support the key finding:

6. Conclusion

Recap of the problem, the solution method and key finding.

7. Acknowledgement if any

8. References