

Numerical Differentiation

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1. Differentiating basis functions

$$f(x) = \sum_i a_i \phi_i(x) \quad \Rightarrow \quad f'(x) = \sum_i a_i \phi_i'(x)$$

2. Finite differences

- ▶ Forward difference
- ▶ Backward difference
- ▶ Centered difference

3. choosing the stepsize h

limitations: if we compute in machine precision ϵ_M , then we cannot expect more than **half** machine precision for numerical differentiation!
Numerical differentiation is an inherently sensitive procedure.

Example: `diffeg1.m`

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Richardson extrapolation

- ▶ A scheme to take two $O(h)$ approximations of $f'(x)$ to make an $O(h^2)$ approximation
- ▶ Example: `diffeg2.m`
- ▶ A sequence acceleration scheme to improve accuracy (and convergence), widely used in other settings.