1. Consider the following cubic polynomial

\[ p(x) = 816x^3 - 3835x^2 + 6000x - 3125. \]

It has three closely spaced roots: 25/15, 25/16, 25/17

(a) Plot \( p(x) \) for 1.43 \( \leq x \leq 1.71 \). Show the location of the three roots.
(b) Starting with the interval [1,2], what does the bisection method do?
(b) Starting with \( x_0 = 1.5 \), what does Newton’s method do?
(c) Starting with \( x_0 = 1 \) and \( x_1 = 2 \), what does the secant method do?

2. Investigate the behavior of the secant method on the function

\[ f(x) = \text{sign}(x - 2)\sqrt{|x - 2|} \]

*Hint: start from “Example Newton 2” from our class website.*

3. Let

\[ f(x_1, x_2) = \frac{1}{2}(x_1^2 - x_2)^2 + \frac{1}{2}(1 - x_1)^2 \]

(a) What is the minimizer of \( f(x_1, x_2) \)?
(b) Compute one iteration of Newton’s method for minimizing \( f(x_1, x_2) \) starting from the point (2, 2). Is this a good step?

4. Let

\[ f(x_1, x_2) = \frac{1}{2}x_1^2 + \frac{9}{2}x_2^2. \]

It’s easy to see that the minimizer is \( x_* = (0, 0) \)

(a) Derive the steepest descent method for finding the minimizer of \( f(x) \).
(b) Compute the first four iterations starting from the point (9, 1).