1. A **proposition** is a statement that is either true (T) or false (F), but not both.

   Standard notation for a proposition: p, q, r, ...

2. New propositions, called **compound proposition**, are formed from existing propositions using logical operations

   Four basic compound propositions:

   (a) **Negation** of p: $\neg p =$ "It is not the case that p"

   (b) **Conjunction** of p and q: $p \wedge q =$ "p and q"

   (c) **Disjunction** of p and q: $p \vee q =$ "p or q" ("inclusive or")

   (d) **Exclusive disconjunction** of p and q: $p \oplus q =$ "this is true either p or q but not both are ture"

   Truth tables display the relationships between the true values of propositions:

p	$\neg p$
T	F
F	T

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

3. (a) The **Implication** $p \to q$ is the proposition "if p then q"

   p is called *hypothesis*, and q is called the *conclusion*.

   Some other common ways of expressing this implications are: "p implies q", "p only if q", "p sufficient for q", and "q necessary for p"

   The proposition $q \to p$ is called the **converse** of $p \to q$.

   The **contrapostive** of $p \to q$ is the proposition $\neg q \to \neg p$.

   (b) **Biconditional** proposition $p \leftrightarrow q$ is the compound proposition "p if and only if q",

   (c) Truth tables of $p \to q$ and $p \leftrightarrow q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

4. More terminology

   (a) A compound proposition is called a **tautology** if that is **always true**, no matter what the truth values of the propositions that occur in it.

   (b) A compound proposition that is **always false** is called a **contradiction**.

   (c) A proposition that is neither a tautology nor a contradiction is called a **contingency**.

   (d) The compound propositions p and q are called **logically equivalent** if $p \leftrightarrow q$ is a *tautology*.

   Notation: $p \equiv q$

   A way to determine whether two propositions are equivalent is to use a truth table.

   Example: Prove De Morgan's law $\neg(p \vee q) \equiv \neg p \wedge \neg q$

5. (a) **Propositional function** $P(x)$: a statement involving the variables x

   The *domain of discourse*, denoted as U, is the set of values x that x is allowed to take in $P(x)$.

   (b) The **universal quantification** of $P(x)$ is the proposition

   "$P(x)$ is true for all value of x in U".

   Notation: $\forall x\ P(x)$, $\forall$ is called the universal quantifier.

   Truth values of $\forall x P(x)$

   "$\forall x P(x)$" = True, when $P(x)$ is true for every x in U.

   "$\forall x P(x)$" = False, when there is an x in U for which $P(x)$ is false.

   (c) The **existential quantification** of $P(x)$ is the proposition

   "There exists an element x in U such that $P(x)$ is true"

   Notation: $\exists x\ P(x)$. $\exists$ is called the existential quantifier.

   Truth values of $\exists x\ P(x)$

   "$\exists x\ P(x)$" = True, when there is an x in U for which $P(x)$ is true.

   "$\exists x\ P(x)$" = False, when $P(x)$ is false for every x in U.