Midterm

Instructions: Please answer the questions succinctly, thoughtfully and legibly. Good luck.

Name:

ID:

<table>
<thead>
<tr>
<th>On section</th>
<th>you got</th>
<th>out of</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>45</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>45</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>30</td>
</tr>
<tr>
<td>Σ</td>
<td></td>
<td>175</td>
</tr>
</tbody>
</table>
1 Definitions [10 points]

1. (4 pts.) Define formally the star (*) operation on languages, i.e. $A^* =$ ...

2. (6 pts.) State the Pumping Lemma for regular languages (formally and completely).

2 A Decision Procedure [15 pts.]

You are given a DFA $D$ and want to determine if it accepts any words of which your name is a substring. Describe a decision procedure to do that.
3 Justified True or False [30 points]

Put an X through the correct box. Provide a brief (but convincing) justification. No credit will be given to correct answers that lack a proper justification. Where appropriate, make your justification a counter-example. Each question in this section is worth 5 points.

1. $(\emptyset \cup \emptyset)^* = \emptyset$

   Explain:

   True  False

2. If $L$ is not a regular language then $L$ is context-free.

   Explain:

   True  False

3. If CFG $G_1$ has fewer rules than CFG $G_2$ then $|L(G_1)| \leq |L(G_2)|$.

   Explain:

   True  False
4. $L = \{a^i b^j | i \geq j \geq 65536\}$ is a regular language.

   Explain:

   True  False

5. Any Context Free Grammar can be transformed into an equivalent grammar having
   one rule of the form $S \rightarrow V_1 V_2 \ldots V_n$ and $n$ rules of the form $V_i \rightarrow a_i, i = 1, \ldots, n$,
   where $V_i$ are variables and $a_i$ are terminals.

   Explain:

   True  False

6. If $\mathcal{T}$ is finite then $L$ is regular.

   Explain:
4  Finite Automata Problems [45 points]

1. (15 pts.) Give a DFA for the language of $(0 \cup 01)^*$.

2. (30 pts.) Determine if the following languages are regular or not and prove your claim in each case.
   a) $\{a^nb^n|n \geq 0\}$
   b) $\{a^5n|n \geq 0\}$
5 CFG Problems [45 pts.]

Write Context Free Grammars for the following languages:

a) (14 pts.) \( A = \{ab^nc^m a^{n+m}|n, m \geq 0\} \)

b) (12 pts.) \( B = \{ab^nc^{n+m}a^m|n, m \geq 0\} \)

c) (7 pts.) \( A \cup B \)

d) (12 pts.) \( A \cap B \)

(Hint: Consider the n’s and the m’s separately)
6 A Minimality Proof [30 points]

a) (10 pts.) Design a DFA for the language \( \{ w \in \{a, b\}^* | w \text{ starts with an } a \text{ and contains an even number of } b's \} \).

b) (20 pts.) Find the smallest DFA for the language above and prove its minimality. (smaller=fewer states.)