

ECS289F — Homework 3

Out Friday, 1/29/10; due in class Monday, 2/8/10

Problem 1. Prove that checking equivalence of CQs is NP-complete, by modifying the proof from class of the same for containment.

Problem 2. A *union of conjunctive queries* (UCQ) \mathbf{Q} is a set $\mathbf{Q} = \{Q_1, \dots, Q_n\}$ of conjunctive queries, with the semantics defined

$$\llbracket \mathbf{Q} \rrbracket^I \stackrel{\text{def}}{=} \bigcup_{i=1}^n \llbracket Q_i \rrbracket^I.$$

- (a) Show that the problems of containment and equivalence for UCQs are easily interreducible.
- (b) Show that the Chandra-Merlin Theorem for containment of conjunctive queries extends to UCQs (you will need to restate it appropriately), and that the complexity of the problem remains the same, i.e., NP-complete.
- (c) Give a procedure for minimizing UCQs and show that, as for CQs, the result of this procedure is unique up to isomorphism, where UCQs \mathbf{Q} and \mathbf{Q}' are *isomorphic* if there is a bijection $h : \mathbf{Q} \rightarrow \mathbf{Q}'$ such that for every CQ $Q \in \mathbf{Q}$, Q is isomorphic to $h(Q)$.

Problem 3. Denote by SPCU (“select-project-cross product-union”) the fragment of the relational algebra where set difference is disallowed.

- (a) Show that any SPCU query can be rewritten equivalently as a union of SPC queries and that this implies that SPCUs are expressively equivalent to UCQs.
- (b) Give an example of an SPCU expression such that the corresponding union of SPC expressions (and hence the corresponding UCQ) is exponentially larger.
- (c) Recall that Π_2^p is the complement of Σ_2^p , the class of languages decidable in nondeterministic polynomial time with access to an oracle for NP. The canonical example of a Π_2^p -complete problem is Q3-SAT (“quantified 3-SAT”): given a Boolean formula $F = B_1 \wedge \dots \wedge B_n$ in conjunctive normal form with three literals in each clause (3-CNF), and a subset X of the variables in F , decide if F is satisfiable for every assignment of the variables in X .

Show that containment of SPCU queries is in Π_2^p .

- (d) **Extra credit.** Show that containment of SPCU queries is Π_2^p -hard.