Buneman Graph

Exercise

1. We will outline the proof of the following theorem.

**Theorem.** The Buneman Graph $\mathcal{B}(H)$, for binary matrix $H$, is a median graph.

To prove the Theorem, first establish the following two facts:

- Show that $\mathcal{B}(H)$ is connected and that the length of the shortest path between two vertices $u, v \in \mathcal{B}(H)$ is equal to the number of components on which $u$ and $v$ differ. Actually, that part was proven in the book in the proof that a Buneman graph is a splits network, so you don’t need to do that.
- Let $v_1, v_2,$ and $v_3$ be any three vertices of $\mathcal{B}(H)$. Consider vertex $v$ such that the $i$th component of $v$ is the block that agrees with the $i$th component of at least two of $v_1, v_2,$ and $v_3$. Show that $v$ is the unique median of $v_1, v_2,$ and $v_3$.

Now complete the proof that the Buneman Graph, given $H$, is a median graph.