0. Rob Gysel will assign some problems related to Chordal graphs - do those.

1. Read the notes on the $O(n^3)$-time algorithm to test if a graph is planar. I believe Algorithm 3.5 on page 80 is correct, but it is not what I had expected. To explain what I had expected, in Step 2b, let $\gamma$ denote the part of cycle $C$ that is not in cycle $C'$ (that notation is also consistent with what is used in the proof of Lemma 3.4).

I had expected that that in Step 2c, the recursive call would be with the graph $P' - \gamma$ and cycle $C'$. That is, the algorithm would trim away the $\gamma$ part of $P'$. Would the algorithm be correct if we did that? If correct, would it be better to do things that way? Fully explain your answer.

2. Explicitly write out the justification that the following are equivalent:
   a. Graph $G$ is $\alpha$-perfect implies $G$ is $\gamma$-perfect.
   b. Graph $G$ is perfect if and only if its complement is perfect.
   c. Graph $G$ is $\alpha$-perfect if and only if it is $\gamma$-perfect.

   (Note that $\gamma$ here has a different meaning than in problem 1.)

3. In the proofs of Lemma 8.1.3 and Theorem 8.1.4 of the notes on perfect graphs, the author uses all three of the terms ‘maximum’, maximum size’, ‘maximal’, when referring to sets.

   Generally, depending on the property of the set, a ‘maximal set’ with some property is not necessarily a ‘maximum size’ set with that property, and sometimes the distinction is critical to an argument. So I don’t use the terms ‘maximal’ and ‘maximum size’ interchangeably.

   Would the proofs of the lemma and the theorem work if every occurrence of ‘maximum’ and ‘maximal’ was changed to ‘maximum size’?

   (OK, yes I see the degeneracy - I don’t mean that ‘maximum size’ should be changed to ‘maximum size size’).

4. Suppose that $G = G_1 \cup G_2$, that $G_1 \cap G_2$ is a clique, and that $G_1$ and $G_2$ are perfect. Prove that $G$ is perfect.