CS 225, Winter 2012: Graph Theory and Algorithms - Dan Gusfield

The field is vast with many deep subareas. What I will try to do in this course is discuss some of the classic gems of graph theory, some interesting algorithms, and some specialized topics in graph theory and graph algorithms of particular interest to me and members of the class. If you have a specialized interest, please let me know and I will see if we can work it in to the class. Two classic topics that we will not cover are algorithmic methods in network flow and bipartite matching because they are already well covered in CS 122A,B and 222A. However, we might discuss some applications or purely combinatorial aspects of network flow and matching, and we will cover some cut problems and methods that don’t use network flow. We will also not spend time on any NP-hardness proofs.

There is no assigned text for the class. The class will draw from several sources, mostly books, listed below, and some papers. I will copy the relevant pages and post them on the class website.

Not having taught this class before, I can’t be sure of how much time each topic will take, so I don’t know that we will cover everything listed, but here is what I hope to cover (at this point).

1. General introduction to graph theory, exposed through Euler paths and tours; Hamilton paths - sufficient conditions; properties of trees; Prufer’s Proof. deBruin graphs. Basic algorithms such as finding all biconnected components, line graphs.

2. Planarity - Euler’s formula, Kuratowski’s theorem, polynomial-time planarity testing algorithm. Other characterizations of planarity. The five color theorem.

3. Perfect graph theory. Chordal graph theory in detail. The clique-tree characterization of chordal graphs; linear-time algorithms to test for a chordal graph through perfect elimination orderings, linear-time algorithms to build a clique-tree of a chordal graph; algorithms to find all minimal separators of a graph. Tree Decomposition. Interval and Permutation graphs. The perfect graph theorem, if I can find an understandable exposition of its proof.

4. Edge and node colorings. Vizing’s theorem.


6. Topics: specialized topics on graphs and algorithms; extreme sets of a graph; approximation methods; random graph theory; expanders; rigidity via matroid theory (if I can understand it in time to teach it); Linear-time
planarity testing (if there is a much simpler exposition than when I studied it 30 years ago); graph realization theory (if I can master this again - I knew it about ten years ago); Cut representations such as cactus graphs and Gomory-Hu Trees; representations of all minimum s-t cuts, etc.

Homeworks: I will try to find challenging problems. Hopefully we will solve an open problem or two.

I am really looking forward to this class. This will not be a ‘Sage on Stage’ kind of class, but rather one where I hope to have very active student participants who follow the class in real time and catch me when I make mistakes. Class participation will be taken into consideration in the course grade.

Some of the books that the course material will be drawn from:

1. Graph Theory – Harary
2. Graph Algorithms – S. Even
3. Graphs and their Uses – O. Ore
4. Intersection graph theory – McKee
5. Algorithmic graph theory and Perfect Graphs – Golombic
6. Introduction to Graph theory – R. Wilson
8. Graph Theory with applications – Bondy and Murty
10. Graph Drawing – Di Battista et al.
12. Randomized algorithms – Motwani