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• PhD in MIT EECS 1996
• Associate Professor of Manufacturing Engineering, Boston University since 1996
  – Stochastic systems
  – Queueing Theory
  – Optimization, pricing and revenue management
    Communication networks, manufacturing systems, supply chains.
John N. Tsitsiklis

- 1983-84 Assistant Professor of EE Stanford.
- Since 1984 Professor of EECS in MIT
  – systems, optimization, control, and operations research. 94 journal papers in these areas.
Congestion-Dependent Pricing of Network Services

- 2000, IEEE/ACM Transactions on Networking
- Fee per call
- Qualitative Properties
  - Time-of-day pricing will often suffice.
  - For many small users near optimality is possible.
System Model

- Service Provider (SP) charging the customer per call.
  - Modem pool with limited capacity (R)
  - Users are charged per connection
  - Price may depend on the current congestion level

- Charging customer relative to the congestion she cause may result in more efficient resource allocation.
Alternatives

- Fixed pricing
- Slowly-varying pricing
- Fluctuating pricing
Similar Problems

• Airline pricing
  – Cost of serving an additional customer is negligible compared to the cost of the flight.
Problem Formulation

- Resource Limited: $\mathcal{R}$
- Each call needs a certain amount of resource: $r$
- Customers are clustered into classes according to the arrival rate ($\lambda$) and holding time ($\mu$): $\mathcal{M}$ classes
- $\lambda_i$ depends on price $u_i$ for class $i$
• Revenue Maximization
  – Monopolistic context

• Welfare Maximization
  – Social welfare
Relevant Models

• Modem pool
• Network provider offering a number of connection types
  • Video;
  • Voice etc.
• Service provider offers multiple types of content
  • Each with different duration and capacity requirement on the server
Assumptions

- Linear constraint on the number of calls that can be accommodated.
  - e.g. BW constraint
- Demand function is known
  - Given the price demand can be calculated
- Charge per call
Optimal Dynamic Policies

- Congestion dependent pricing
  - Finite State
  - Continuous

- The process $\mathcal{N}(t)$ is continuous time Markov chain.
  - Can be uniformized to the form:
    $$ J^* + h(\mathcal{N}) = \max[...] $$
Optimal Dynamic Policies (cont’d)

• Exponential complexity in the number of class $\mathcal{M}$

• Feasible for small $\mathcal{M}$
Implications

- Having more free resources is better
- If there were no resource limitations then:
  
  There is a price $u_\infty$ independent of the state with the property $u_\infty \leq u^*(N)$.

  There is an optimal revenue $I_\infty$ with $I^* \leq I_\infty$.

- Also

  $u^*(n-1) \leq u^*(n)$
An example

- Demand function:
  \[ \lambda(u) = 60 - 5u \]

- \( R = 30 \)
- \( M = 1 \)
- \( \mu = 1 \)
An Example
Static Pricing

• Price is independent of the state
  – Calls may be blocked if resources not available

However,
  – Computation of dynamic optimal price is expensive;
  – Dynamic price discourages the user.
Limiting Regimes

• Many small users
  • \( r << \mathcal{R} \)

• Using static pricing approximate optimality can be reached if the system is large
Revenue Maximization

- Maximize \( u_i \lambda_i(u_i) \)

s.t. \( \sum_i \lambda_i(u_i) r_i / \mu_i \leq R \)
Welfare Maximization

- Single price for volume,
- Does not depend on the demand function.
Light or Heavy Traffic

• Light Traffic
  • All available resources are not used, therefore $u_\infty$ is the optimal price

• Heavy Traffic
  • All available resources are almost always therefore price slightly less than $u_{\text{max}}$ is optimal.
Approximating Static Policies

- Computation of static policies can be expensive if number of classes is large.
- To approximate:
  - We can use the price suggested by the non-linear optimization problem.
    - Tune $q$
    - Tune $u_i$
Examples, Single Class

- $R=30$, $M=1$, $r=1$, $\lambda(u) = 60 - 5u$

<table>
<thead>
<tr>
<th>$\lambda_0$</th>
<th>$\lambda_1$</th>
<th>$\mu$</th>
<th>$\rho \triangleq \frac{\lambda_0}{R\mu/r}$</th>
<th>$J_s$</th>
<th>$J^*$</th>
<th>$J_{ub}$</th>
<th>$\frac{J^<em>-J_s}{J^</em>} \times 100%$</th>
<th>$u_{ub}$</th>
<th>$u_s$</th>
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<tbody>
<tr>
<td>45.0</td>
<td>5.0</td>
<td>1.0</td>
<td>1.5</td>
<td>99.43</td>
<td>99.82</td>
<td>101.25</td>
<td>0.39%</td>
<td>4.5</td>
<td>4.8</td>
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<td>5.0</td>
<td>1.0</td>
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<td>1.0</td>
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<td>7.12</td>
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<td>0.5</td>
<td>3.0</td>
<td>78.3</td>
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<td>6.0</td>
<td>6.16</td>
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<tr>
<td>50.0</td>
<td>5.0</td>
<td>0.5</td>
<td>3.33</td>
<td>91.12</td>
<td>92.49</td>
<td>105.00</td>
<td>1.48%</td>
<td>7.0</td>
<td>7.06</td>
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<td>45.0</td>
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<td>0.2</td>
<td>7.5</td>
<td>41.57</td>
<td>42.03</td>
<td>46.80</td>
<td>1.09%</td>
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<td>7.51</td>
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<td>45.0</td>
<td>0.5</td>
<td>0.1</td>
<td>15</td>
<td>230.60</td>
<td>232.18</td>
<td>252.00</td>
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<td>84.0</td>
<td>81.0</td>
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<td>45.0</td>
<td>5.0</td>
<td>0.05</td>
<td>30</td>
<td>12.23</td>
<td>12.28</td>
<td>13.05</td>
<td>0.41%</td>
<td>8.7</td>
<td>8.45</td>
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</table>
Responsiveness
Two Class example

- $R = 155, r_1 = 4, r_2 = 1, \mu_1 = 1, \mu_2 = 2, \lambda_i(u_i) = \lambda_{0,i} - u_i\lambda_{1,i}$

<table>
<thead>
<tr>
<th>Case</th>
<th>$\lambda_{0,1}$</th>
<th>$\lambda_{1,1}$</th>
<th>$\rho_1$</th>
<th>$\lambda_{0,2}$</th>
<th>$\lambda_{1,2}$</th>
<th>$\rho_2$</th>
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<td>Case 1</td>
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<td>4</td>
<td>1.032</td>
<td>350</td>
<td>35</td>
<td>1.129</td>
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<td>Case 2</td>
<td>40</td>
<td>4</td>
<td>1.032</td>
<td>500</td>
<td>50</td>
<td>1.613</td>
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<td>Case 3</td>
<td>80</td>
<td>8</td>
<td>2.064</td>
<td>350</td>
<td>35</td>
<td>1.129</td>
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<tr>
<td>Case 4</td>
<td>80</td>
<td>8</td>
<td>2.064</td>
<td>500</td>
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<td>1.613</td>
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<td>1280</td>
<td>128</td>
<td>4.129</td>
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<td>256</td>
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<td>Case 7</td>
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<td>64</td>
<td>16.512</td>
<td>5120</td>
<td>512</td>
<td>16.516</td>
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<tr>
<td>Case</td>
<td>$J_s$</td>
<td>$J^*$</td>
<td>$J_{ub}$</td>
<td>$\frac{J^<em>-J_s}{J^</em>} \times 100%$</td>
<td>$u_{ub,1}$</td>
<td>$u_{s,1}$</td>
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<tr>
<td>Case 1</td>
<td>945.79</td>
<td>952.63</td>
<td>972.85</td>
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<td>1281.65</td>
<td>1317.32</td>
<td>0.88%</td>
<td>7.62</td>
<td>8.74</td>
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<td>Case 3</td>
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<td>977.28</td>
<td>1012.43</td>
<td>1.22%</td>
<td>7.71</td>
<td>8.23</td>
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<tr>
<td>Case 4</td>
<td>1273.9</td>
<td>1288.97</td>
<td>1329.72</td>
<td>1.17%</td>
<td>8.7</td>
<td>9.26</td>
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<td>Case 5</td>
<td>2206.1</td>
<td>2235.13</td>
<td>2349.22</td>
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<tr>
<td>Case 6</td>
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<td>2724.60</td>
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<td>10</td>
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<td>Case 7</td>
<td>2804.1</td>
<td>2820.47</td>
<td>2912.30</td>
<td>0.58%</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>
System Occupancy
Conclusion

- Static pricing can be very close to optimal
- Demand controls the price