Simplified Master Theorem

A recurrence relation of the following form:

\[ T(n) = \begin{cases} 
  c & n < c_1 \\
  aT(n/b) + \Theta(n^i), & n \geq c_1 
\end{cases} \]

Has as its solution:

1) If \( a > b^i \) then \( T(n) = \Theta(n^\log_b a) \) (Work is increasing as we go down the tree, so this is the number of leaves in the recursion tree).

2) If \( a = b^i \) then \( T(n) = \Theta(n^i \log_b n) \) (Work is the same at each level of the tree, so the work is the height, \( \log_b n \), times work/level).

3) If \( a < b^i \) then \( T(n) = \Theta(n^i) \) (Work is going down as we go down the tree, so dominated by the initial work at the root).