Problem Set 3—Due Friday, February 16, 3PM

(30) **Problem 1.** The *yes/no clique*— problem is: given an undirected graph $G=(V,E)$ and a target integer $k$, is there a clique of size $k$? A *clique* is a set of vertices $C$ in $V$ such that each pair of vertices $(u,v)$ in $C$, is also an edge in $E$ (thus every pair of vertices in a clique are connected by an edge).

a) Show that the *yes/no clique* problem is in NP (note, this problem is NP-C but you are NOT being asked to prove that).

b) Show that you can use a program which solves the *yes/no clique* problem to actually find a clique of size $k$ (when one exists). You should find the clique using a polynomial number of calls to the *yes/no clique* routine, plus polynomial additional work. Thus you are showing that the problem of finding a clique of a given size is polynomially reducible to *yes/no clique*.

c) Give a polynomial-time algorithm for *yes/no clique*— when $k < c$ for a constant $c$. Would your algorithm still run in polynomial time if we restrict $k$ so $k < \log n$, with $n$ the number of vertices in the graph?

(27) **Problem 2.** Consider the directed *Hamilton Path* problem (DHP) described on page 979.

a) Show that DHP $\leq_p s,t$ Hamilton Path where $s,t$ Hamilton Path takes as input a directed graph $G=(V,E)$ and two designated vertices in $V$, and returns yes when there is a Hamilton path starting at $s$ and ending at $t$.

b) Consider the problem of finding a shortest *simple* path in a directed graph (with negative cycles) from $s$ to $t$ (thus you are given a directed graph $G$, edge weights, possibly negative, and two designated vertices). Show that this problem is NP-hard (hint: Use your result of part a)).

(18) **Problem 3.** Suppose we are given a *Traveling Sales Man* (TSP) problem where cities are points in the plane and distances are actual Euclidean distances (Call this the Euclidean TSP).

Show that the proof of theorem 34.14 p. 1013 in the text does not prove that the Euclidean TSP is NP-hard (this problem is in fact still NP-hard, but you are not asked to prove this).