Problem Set 4 – Due Tuesday, April 28, 2009: 3:15

1. [10] is the following true or false (justify your answer): \( A = \emptyset \iff \mathcal{P}(A) = \emptyset \)

2. [24] Which of the following conditions imply that \( B = C \)? In each case, either prove or give a counterexample.
   (a) \( A \cup B = A \cup C \)
   (b) \( A \cap B = A \cap C \)
   (c) \( A \oplus B = A \oplus C \)

3. [24] Suppose that \( A, B \) and \( C \) are sets. For each of the following statements either prove it is true or give a counterexample to show that it is false.
   (a) \( (A \setminus B) \cup C = (A \setminus B) \cup (A \setminus C) \)
   (b) \( (A \setminus B) \times C = (A \times C) \setminus (B \times C) \)
   (c) \( (A \oplus B) \times C = (A \times C) \oplus (B \times C) \)

4. [10] Write a regular expression for the language that is the set of all nonempty strings over \( \{a, b\} \) that start and end with different characters. Make your regular expression as short as you can.

5. [12] Write a regular expression for the language that is the set of all strings over \( \{a, b, c\} \) that have exactly two ”c”’s. Make your regular expression as short as you can.

6. [14] Suppose \( L_1 \) and \( L_2 \) are regular languages, then is \( L_1L_2 \) also a regular language? Justify your answer.